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TORSION, BENDING, AND SHEAR IN RECTANGULAR PRESTRESSED CONCRETE BEAMS

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "TORSION, BENDING, AND SHEAR IN RECTANGULAR PRESTRESSED CONCRETE BEAMS," submitted by Erik Bech Jacobsen in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

This study followed a continuing program of research carried out at the Structural Laboratory of the University of Alberta by Dr. J. Warwaruk (*). The results of the entire program will be presented as a report at a later date.

This phase of the investigation was designed to achieve a better understanding of the behavior of prestressed rectangular concrete beams subjected to bending, torsion, and shear.

Twenty-two beams having a nominal cross section of 6 x 12 in. and containing mild steel web reinforcement were tested. Fourteen beams were concentrically prestressed, and eight were prestressed eccentrically.

The testing equipment used for this investigation allowed independent application of the twisting moment and transverse loads. This permitted the specimens to be tested in varying ratios of twisting moment to bending moment. All beams were tested to failure by applying load in a series of predetermined increments.

The test results are presented in the form of tables, graphs, and interaction diagrams.

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CHAPTER I

INTRODUCTION

1-1 INTRODUCTORY REMARKS

Until the last decade the problem of torsion and its effect on the behavior of prestressed concrete members was largely ignored. This oversight stemmed partly from the infrequency of torsionally loaded members in the structures of the day. In more recent practise the trend has been toward less conventional and more graceful architectural forms in which the effect of torsion often is a primary factor governing the design. Hence an obvious need existed for furthering the state of knowledge with regard to torsion and its effect on prestressed members. It was to satisfy this need that the research program on torsion which is still being continued today was initiated.

Presently, studies have already been completed dealing with the interaction of torsion and shear in prestressed members. As well, research is continuing in the field of combined loadings of bending, torsion, and shear. This loading case is commonly found in most structures. The aim of this particular investigation is to attempt to supplement the data which has already been obtained in both of these fields, namely the interaction of torsion and bending as well as the interaction of torsion, bending, and shear.

1-2 OBJECT

The main objective of this investigation was to study the behavior of prestressed rectangular concrete beams subjected to a combined loading of torsion and bending as well as torsion, bending, and shear. Included in the study were variables such as the type and level of prestress, spacing of transverse reinforcement, presence of shear, and the proportion of twisting moment relative to bending moment.

As nearly as possible it was attempted to stress all the prestressing strands to the same level of prestress. In order to enable the calculation of the effective prestress force at the time of testing to be performed, the elastic shortening of the concrete upon release, as well as the time dependent strains, were measured.

During the testing of each specimen the behavior of the non-prestressed transverse reinforcement was monitored by means of electrical resistance strain gauges located at suitable intervals in the gauge length.

All the beams reported on were fabricated and tested according to the procedures outlined in Chapter III. The results of the tests are summarized in tables, graphs, and discussions.

1-3 SCOPE

The test specimens in this investigation were divided into four groups. The first group consisted of Beams 301-307, the second, 321-327, the third, V301-V307, and the fourth group consisted of Beams V302P and V322P. All beams had a nominal cross section of 6x12 inches and an overall length of 10'-1", and were all prestressed using high

strength steel strand. The physical properties of all the specimens are presented in Table 3.1.

A description of the testing equipment used is given in Chapter III. This equipment allowed independent application of the twisting moment and the transverse load so that the beams could be tested in pure torsion or bending, or in varying ratios of torsion to bending as well as torsion, bending, and shear.

The results of the tests are summarized in Tables 4.1 and 4.2. In addition, Moment-Deflection curves, Torque-Twist curves, Dimensional and Non-Dimensional Interaction Diagrams, and other tables and discussions are also presented.

CHAPTER II

REVIEW OF PREVIOUS RESEARCH

2-1 INTRODUCTION

Within the last ten years a considerable amount of research has been performed in the field of torsion applied to prestressed concrete beams. The initial stages of these investigations involved only the effect of pure torsion, whereas recently the research effort has been shifted to encompass combined loadings of torsion, shear, and bending. The latter loading condition is more realistic in that it corresponds closely to the loadings which building members are subjected to.

A fairly extensive review of research performed in these areas has been presented by Stark (1). The review covered in this chapter is hence intended to be only of a supplementary nature, and includes literature published in the intervening period.

2-2 PREVIOUS RESEARCH

2-2.1 BISHARA

Bishara (2) tested twenty-four pretensioned prestressed concrete beams. These tests were intended to study experimentally the behavior, ultimate capacity, and rupture criteria of beams with web reinforcement subject to combined loadings of bending, torsion, and shear. He stated

that the simultaneous action of torque, moment, and shear reduced the capacities of the members below their corresponding capacities under the action of moment and shear only. It was observed that for a constant moment to shear ratio the reduction percentage was closely related to the moment to torque ratio. The lower the ratio of moment to torque the higher was the reduction percentage.

Bishara also found that for relatively low values of moment to torque ratios, the torque capacity of a member increased with an increase in this ratio. After the maximum torque capacity was reached any increase in the moment to torque ratio resulted in a subsequent decrease in the torsional capacity. In addition, he concluded that the torsional strength of prestressed members could be increased beyond their pure torsional strength by the simultaneous action of bending and shear. This increase in strength for a rectangular beam in some cases was as much as 40%.

In the elastic range it was found that the torsional stiffness constant of the flanged members tested could be evaluated satisfactorily by the use of St. Venant's elastic torsion theory. For the rectangular members tested these measured torsional stiffness constants were usually lower than the theoretical values. Once cracking occurred, however, the St. Venant elastic torsion theory was no longer applicable, and neither the cracking load nor the ultimate capacity of the members could be satisfactorily predicted by this method.

2-2.2 MUKHERJEE AND WARWARUK

Mukherjee and Warwaruk (3) reported on the behavior of web

reinforced prestressed beams under combined loading. In all a total of fifty-two beams were tested of which twenty-eight were subjected to bending and torsion only, and the remaining twenty-four were tested under combinations of bending, torsion, and shear. Their findings closely agreed with those of Bishara mentioned in 2-2.1. Mukherjee and Warwaruk in addition stated that an increase in prestress caused corresponding increases in cracking and ultimate torques of prestressed beams, whereas the ultimate twist of such beams was decreased. The non-prestressed reinforcement provided in both directions prevented a brittle type failure and improved the ductility of the beams. Another conclusion made was that the cracking torque of the members was little affected by the non-prestressed reinforcement, but the ultimate strength in torsion was definitely increased by the addition of longitudinal and transverse reinforcement. For a loading combination of bending and torsion only, the transverse reinforcement generally yielded at failure with the exception of beams tested under small torque to bending moment ratios.

They also concluded that the beams tested basically failed in three different modes. Under combined loading, failure occurred either by crushing of the concrete on top of the member, or along an indirect plane on the vertical faces. A third mode of failure, characterized by a compression zone at the bottom, could become critical for eccentrically prestressed beams which were subjected to relatively large ratios of torque to bending moment.

For any combination of torque to bending, the torsional strength of the beams was found to be reduced by the presence of shear.

2-2.3 ZIA AND GANGARAO

Zia and GangaRao (4) tested twenty-eight concentrically prestressed beams and fourteen eccentrically prestressed beams, all subjected to various combinations of bending and torsion. They found that the behavior of beams prior to cracking of the concrete was unaffected by the reinforcement provided. After cracking, the behavior depended primarily on the amount of reinforcement and on the ratio of bending to torque. As the ratio of bending to torque was increased, the difference between the cracking and ultimate capacities of specimens increased, and the rotation and deflection of the specimens also became greater at the ultimate load. With regard to the amount of transverse reinforcement provided, the authors found that a reduction in the stirrup spacing improved both the ductility and the ultimate strength of the beams. Other factors which led to improved ductility was eccentric prestressing and the provision of longitudinal mild steel.

Zia and GangaRao also concluded that the cracking torque of a specimen was significantly increased by increases in the level of prestress. Furthermore, increasing the level of prestress led to an increase in the stiffness of a specimen prior to cracking. With the same amount of prestressing force the cracking moment capacity of eccentrically prestressed specimens was reduced slightly as compared to concentrically prestressed members.

Increasing the bending to torque ratios was found to increase the ultimate torsional capacity of the specimens under combined loads. These increases ranged from twenty to fifty percent of the pure tor-

sional capacity of the specimens.

CHAPTER III

TEST SPECIMENS, FABRICATION, EQUIPMENT, INSTRUMENTATION, AND PROCEDURE

3-1 TEST SPECIMENS

In this study twenty-two beams were tested. With the exception of two, the beams were all provided with transverse reinforcement in the form of vertical stirrups. Two beams contained no nonprestressed reinforcement. The total series of beams was divided into four groups as outlined in Table 3.1. Groups I and II, consisting of Beams 301 to 307 and 321 to 327, were tested in combined bending and torsion. From each group one beam was subjected to bending only, and one to torsion only. Groups III and IV were tested in combined bending, torsion, and shear. One beam from Group III was tested in shear and bending only. All beams had a nominal cross section of 6 x 12 inches, and their overall length was 10' - 1".

3-1.1 CONCRETE

The mix design used in the fabrication of each specimen in this study was of the following proportions:

(1)	CEMENT (TYPE III)	150 Lbs.
(2)	SAND	310 Lbs.
(3)	COARSE AGGREGATE	500 Lbs.

The amount of water used averaged 85 lbs. per batch. This mix

yielded seven cubic feet of concrete with an approximate slump of 3 inches.

3-1.2 SAND

A sieve analysis of the sand used is given in Table 3.2. The average moisture content of the sand was 4%.

3-1.3 COARSE AGGREGATE

The coarse aggregate used was 3/4" maximum size crushed rock with an average moisture content of 1.7%. The sieve analysis for this aggregate is presented in Table 3.3.

3-1.4 REINFORCEMENT

The transverse reinforcement used in the test specimens is described in Table 3.1. The #3 deformed bars were from two different heats designated Type A and Type B. The #2 plain bars were from one heat only, designated as Type F. Representative samples of Type A, Type B, and Type F bars were subjected to tension tests in order to obtain the stress-strain curves shown in Figure 3.8. The arrangement of the non-prestressed reinforcement is illustrated in Figures 3.1, 3.2, and 3.3. Larger amounts of transverse reinforcement was provided in the areas outside the gauge length in order to ensure that failure would not occur in this region.

3-1.5 PRESTRESSING STEEL

The cable used for prestressing the test specimens was 3/8" and 1/2" diameter - 7 wire strand with a guaranteed minimum yield strength of

SERIES	BEAM NO.	CONCRETE STRENGTH f'_c p.s.i.	f'_{sp} p.s.i.	PRESTRESSING REINFORCEMENT DESCRIPTION	P_p %	f_{up} k.s.i.	TRANSVERSE REINFORCEMENT DESCRIPTION	P_t %	f_{yt} k.s.i.	EFFECTIVE PRESTRESS P Kips	$\frac{e}{d}$	$\frac{\sigma}{f'_c}$
I	301	5148	569	4 - 1/2" Dia. 7 Wire Strand 2 - 3/8" Dia. 7 Wire Strand Total Area = 0.735 sq. in.	1.02	250.0	#2 Rectangular Closed Stirrups @ 3-1/2" Area Of One Leg = 0.05 sq. in.	0.60	55.5	102.20	0	0.28
	302	4310	380						55.5	103.30		0.33
	303	3896	394						55.5	99.12		0.35
	304	4255	433						55.5	95.78		0.31
	305	4496	380						55.5	97.11		0.30
	306	4983	464						55.5	100.51		0.28
	307	4950	495						55.5	100.35		0.28
II	321	4791	486		1.02	250.0		0.60	55.5	101.59	1/6	0.29
	322	4573	394						55.5	94.71		0.29
	323	5168	471						55.5	95.63		0.26
	324	4056	394						55.5	94.76		0.32
	325	4432	438						55.5	95.96		0.30
	326	4667	462						55.5	99.98		0.30
	327	4703	451						55.5	100.30		0.30
III	V301	5115	517		1.02	250.0		0.60	55.5	105.13	0	0.29
	V302	4821	486						55.5	102.66		0.30
	V303	5092	486						55.5	101.78		0.28
	V304	5009	433						55.5	105.08		0.29
	V305	4620	429						55.5	105.45		0.32
	V307	5056	531						55.5	103.78		0.28
IV	V302P	4750	482					0		100.43		0.29
	V322P	4644	398							97.59		0.29

TABLE 3.1 PROPERTIES OF TEST SPECIMENS

SIEVE SIZE	WEIGHT RETAINED (gms.)	% RETAINED	CUMULATIVE % RETAINED	A.S.T.M. STANDARD
# 4	17.5	3.0	3.0	0 - 5
# 8	85.2	14.7	17.7	
# 16	54.6	9.5	27.2	20 - 55
# 30	60.0	10.3	37.5	
# 50	208.4	35.8	73.3	70 - 90
#100	122.9	21.1	94.4	90 - 98
PAN	17.8	3.1	-	
SILT	14.4	2.5	-	
TOTAL	580.8	100.0	253.1	
FINENESS MODULUS 2.53				

TABLE 3.2 SIEVE ANALYSIS OF SAND

SIEVE SIZE	WEIGHT RETAINED (lbs.)	% RETAINED	CUMULATIVE % RETAINED
3/4"	0.30	1.1	1.1
3/8"	15.63	58.4	59.5
# 4	10.03	37.5	97.0
PAN	0.80	3.0	100.0
TOTAL	26.76	100.0	

TABLE 3.3 SIEVE ANALYSIS OF COARSE AGGREGATE

250 Ksi. A representative sample of each size was subjected to tension tests in order to obtain the stress-strain curves shown in Figure 3.7.

3-2 FABRICATION

As the first step in the fabrication of the beams, the prestressing cables were cut and pulled into place between two concrete bulkheads. These bulkheads were fastened to the laboratory floor by eight high-strength bolts. At the north bulkhead load cells and wedge-grip end anchorages were installed. This arrangement is illustrated in Figure 3.5. The south bulkhead, shown in Figure 3.6, served as the point of force application, and again wedge-grip end anchorages were used to lock the prestressing cables in place.

Once the system was properly aligned, each cable was stressed individually using a Simplex center-hole hydraulic jack operated by an electric pump. Although it was attempted to stress each cable to the same level of prestress, small variations in end anchorage losses made this virtually impossible.

The transverse reinforcement was then placed by wiring the ties to the prestressing cables at the desired locations. In the areas outside the gauge length additional reinforcement in the form of longitudinal and inclined bars was also provided. At strain gauge locations the #2 plain bars were ground smooth, and type A-7 SR-4 electrical resistance strain gauges were attached. These strain gauges were then waterproofed with three applications of GW-2 waterproofing compound and wrapped with electrical tape to ensure their safety during casting.

The steel forms were then cleaned and oiled, and placed in

position between the concrete bulkheads. As the final step prior to casting the forms were bolted securely into place.

The actual mixing of the concrete was performed in the laboratory using a nine cubic foot capacity mixer. Each batch of concrete contained the exact amount of materials by weight as mentioned in section 3-1.1, with one batch being sufficient for each beam including its control cylinders. The concrete was thoroughly mixed and the water content proportionately adjusted until a 3 inch slump was obtained. The concrete was then deposited in the forms with the aid of an internal vibrator which was used for compaction purposes.

With each specimen five six by twelve inch control cylinders were also cast which were cured and stored under identical conditions to the beams. Of these, three cylinders were tested in compression and the remaining two in tension. Before performing the compression tests, each cylinder was capped with a compound of sulphur and fire clay. These tests were all performed on the same day as the corresponding beam test.

The day following casting, the steel forms were removed and the beams and test cylinders covered with moist burlap and plastic for an additional six days. At this time the plastic and burlap was removed and final readings were taken on each load cell. This was done in order to determine the force exerted by each prestressing cable.

Mechanical gauge points were positioned on both vertical faces of the beams, and initial readings were taken using an 8 inch DEMEC deformation gauge. The location of the gauge points are shown on Figure 3.4.

Prior to cutting the cables, heat from a cutting torch was applied uniformly over a length of about two feet, thus allowing a gradual

transfer of stress from the cables to the concrete. Once the prestress was released a second set of readings was taken on all the gauge points, enabling the instantaneous elastic shortening of the concrete to be calculated. The beams and control cylinders were then set aside for additional air curing until the day of testing. Moist curing was not deemed necessary in this period since high-early strength cement was used.

3-3 TEST EQUIPMENT

The arrangement of the equipment used for testing the beams is illustrated in Figure 3.9 and Figure 3.10. Two load application systems were used such that the torsional and the bending moment could be applied to the beams independently. The transverse load was supplied by a 100-Kip Amsler jack, this load being transmitted to a distributing beam supported by rollers at each end. Two such rollers rested on a roller assembly which in turn rested on a steel collar fastened to the specimen being tested. This system allowed the specimen to twist freely, permitting the torsional moment to be distributed evenly along the member.

The east end of each member was supported by the twisting head through which the torsional moment was applied. An illustration of the twisting head is given in Figure 3.11. This apparatus permitted the specimen to rotate about its longitudinal axis as well as about both a horizontal and a vertical axis perpendicular to the longitudinal axis of the beam.

The forces in the cables which applied the torsional moment to the twisting head were produced by the system shown in Figure 3.12.

These forces were controlled by means of a hand operated hydraulic jack and were measured using load cells and a Baldwin Lima Hamilton strain indicator.

At the west end the specimen was supported by the fixed head as illustrated in Figure 3.13. The fixed head permitted translation of the specimen in a direction parallel to its longitudinal axis as well as rotation about a horizontal axis perpendicular to the longitudinal axis of the beam.

3-4 INSTRUMENTATION

3-4.1 ANGLE OF TWIST

The angle of twist through which a member rotated when subjected to torsional moment was measured by two twistmeters. The location of these is shown in Figures 3.1,3.2, and 3.3. Each twistmeter consisted of an elbow-type aluminum bracket, pin jointed at one end and supported at the other by a micrometer screw. On top of the 1" x 1-1/2" bracket a spirit level was mounted thus enabling any rotation of the beam to be detected by observing the displacement of the bubble. This assembly was attached to the top face of the beam by means of a clamping bracket. The angle of twist through which each twistmeter rotated was then directly proportional to the difference in micrometer readings between successive load increments. Knowing the distance between the two twistmeters, the rotation of the beam in radians per unit of length could be computed.

3-4.2 DEFLECTIONS

The deflection of the beams were obtained at the locations shown in Figures 3.1, 3.2, and 3.3. At each of these locations U-shaped steel brackets were clamped to the beam from which two freely hanging scales were suspended on both vertical faces of the beam. The vertical movement of each scale was observed using precise levels, one on each side of the beam. The deflections were measured to the nearest hundredth of an inch, with the average of the two readings indicating the deflection of the beam.

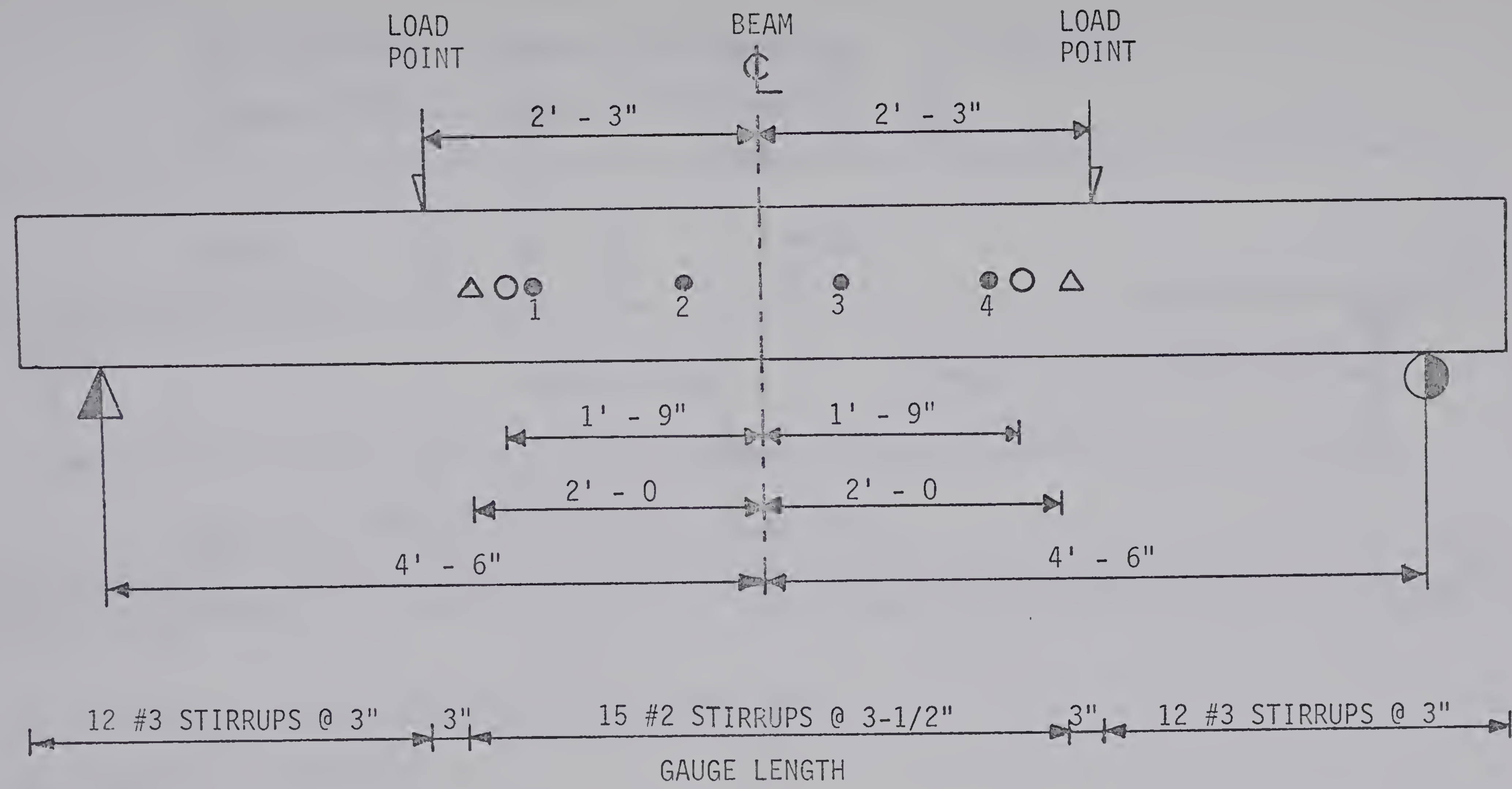
3-4.3 REINFORCEMENT STRAINS

The reinforcement strains were measured using a Budd Type: P-350 strain indicator coupled to a switching and balancing box. The strain gauges were hooked up to the switching and balancing box together with a compensating gauge of the same gauge factor. This compensating gauge, mounted on a piece of steel, was waterproofed and buried in a concrete cylinder in order to simulate the conditions of the actual measuring gauges.

3-5 TESTING PROCEDURE

Immediately before placing the test specimen in the testing apparatus final readings were taken on the mechanical gauge points. This permitted the calculation of the loss in prestress force due to the time dependent strains which had occurred from the time of release of prestress up to the time of testing.

The specimen was then placed in position and tested to failure through the application of a series of predetermined load increments. The increments in the torsional moment and the transverse load were applied simultaneously in magnitudes dependent upon the ratio of torsional moment to bending moment. Whenever possible these increments were reduced near the ultimate capacity of the specimen enabling more data to be collected in this range. For each load increment all the instrumentation was read and the crack pattern marked. Ultimately the specimen was tested to failure, and observations on the failure mechanism were made. It should be noted here that at no time did a specimen fail outside the gauge length.

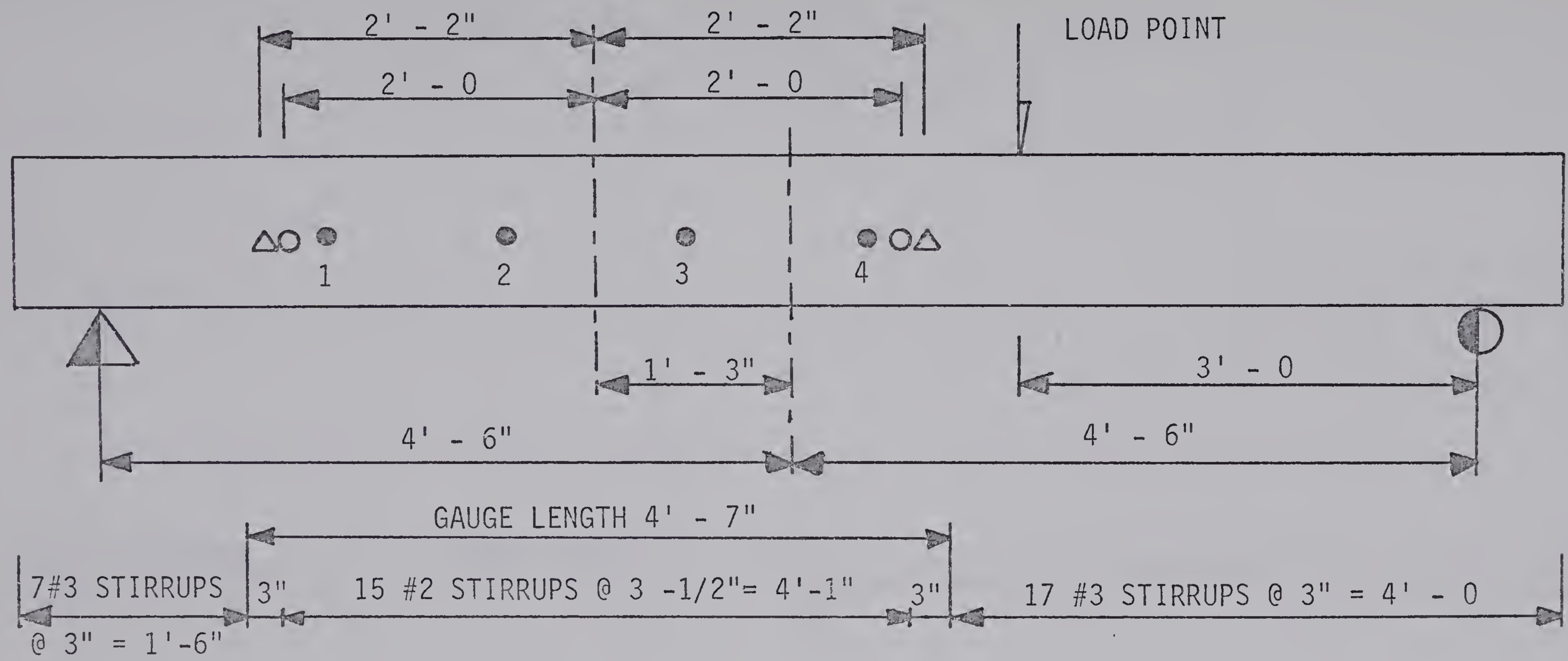


△ LOCATION OF TWO DEFLECTION GAUGES (ONE EACH SIDE)

○ LOCATION OF TWISTMETER

● STRAIN GAUGE LOCATION (SR-4 ELECTRICAL)
1,2,3, & 4 - STIRRUPS

FIGURE 3.1 TYPICAL SPECIMEN (BEAMS OF GROUPS I & II)

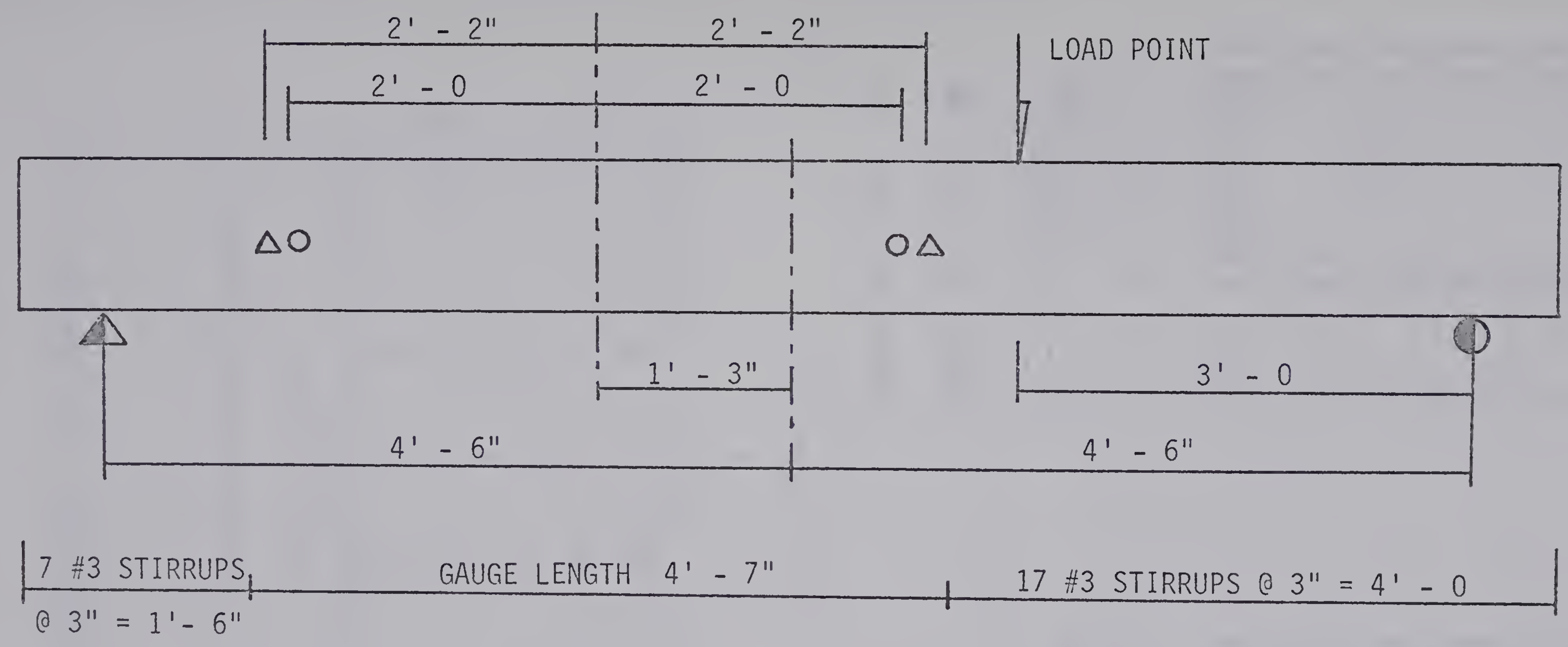


△ LOCATION OF TWO DEFLECTION GAUGES (ONE EACH SIDE)

○ LOCATION OF TWISTMETER

● STRAIN GAUGE LOCATION (SR-4 ELECTRICAL)

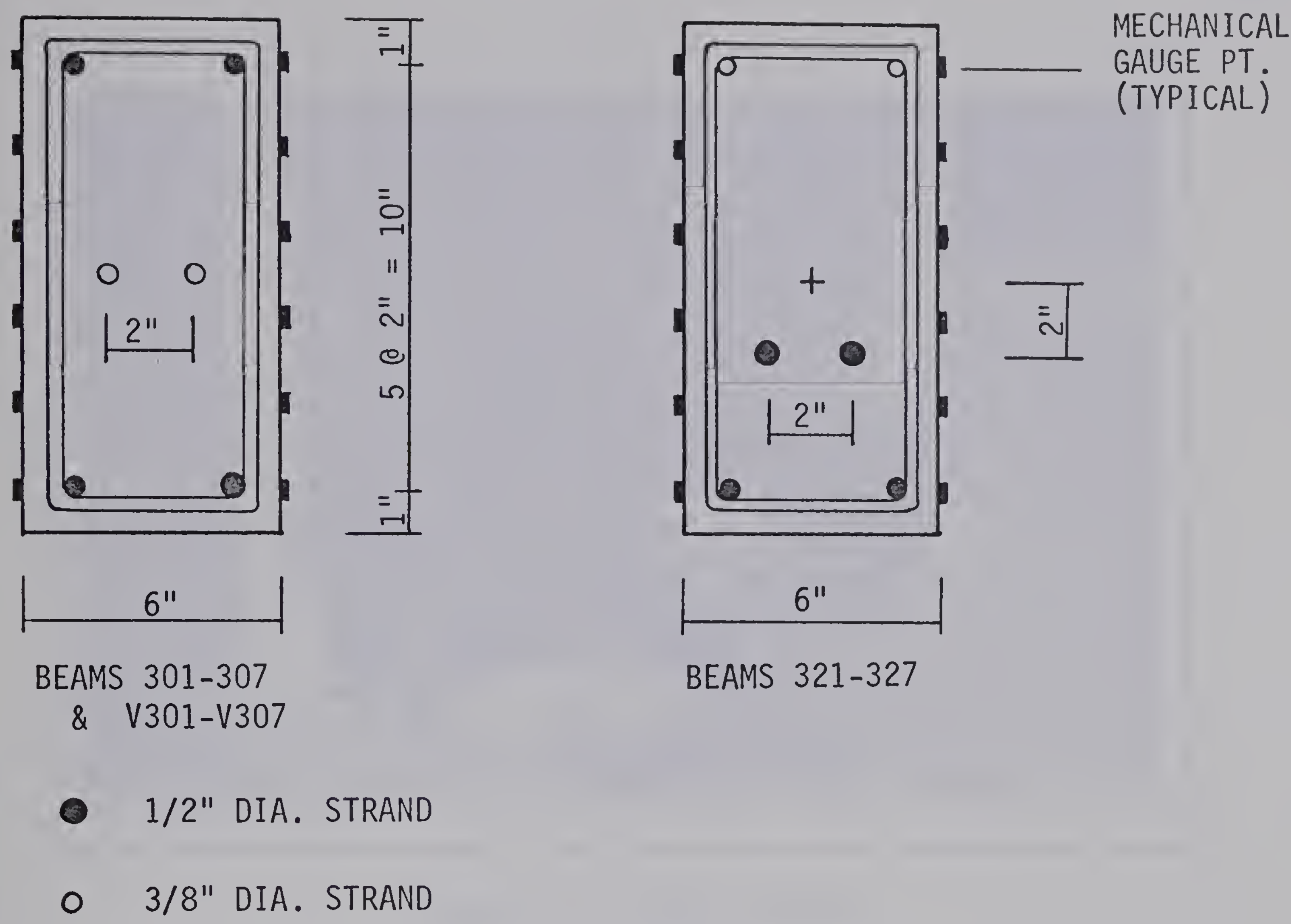
FIGURE 3.2 TYPICAL SPECIMEN (GROUP III BEAMS)



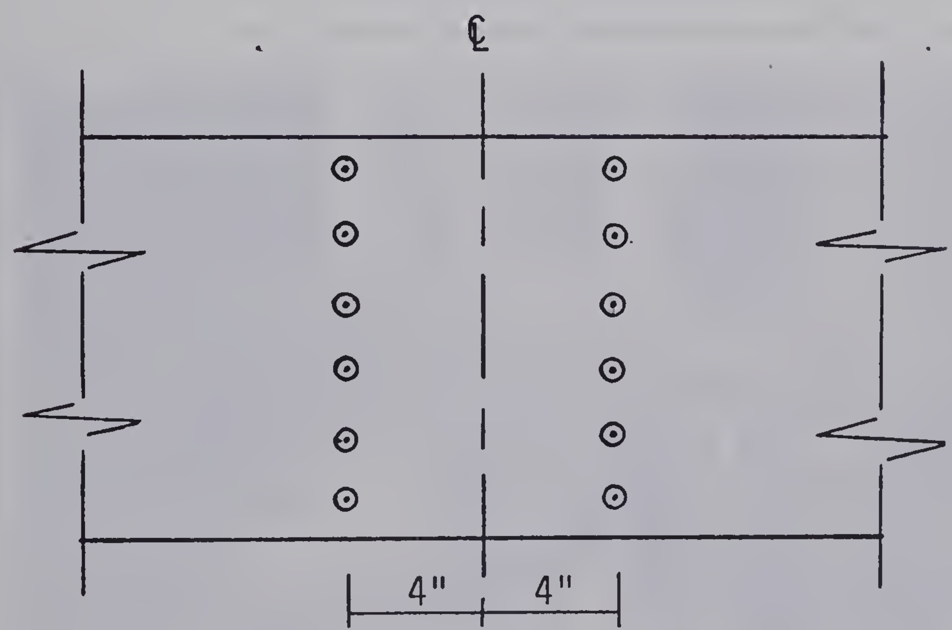
△ LOCATION OF TWO DEFLECTION GAUGES (ONE EACH SIDE)

○ LOCATION OF TWISTMETER

FIGURE 3.3 TYPICAL SPECIMEN (GROUP IV BEAMS)



(a) Cross Sections



(b) Side View Showing Mechanical Gauge Points

FIGURE 3.4 BEAM SECTIONS

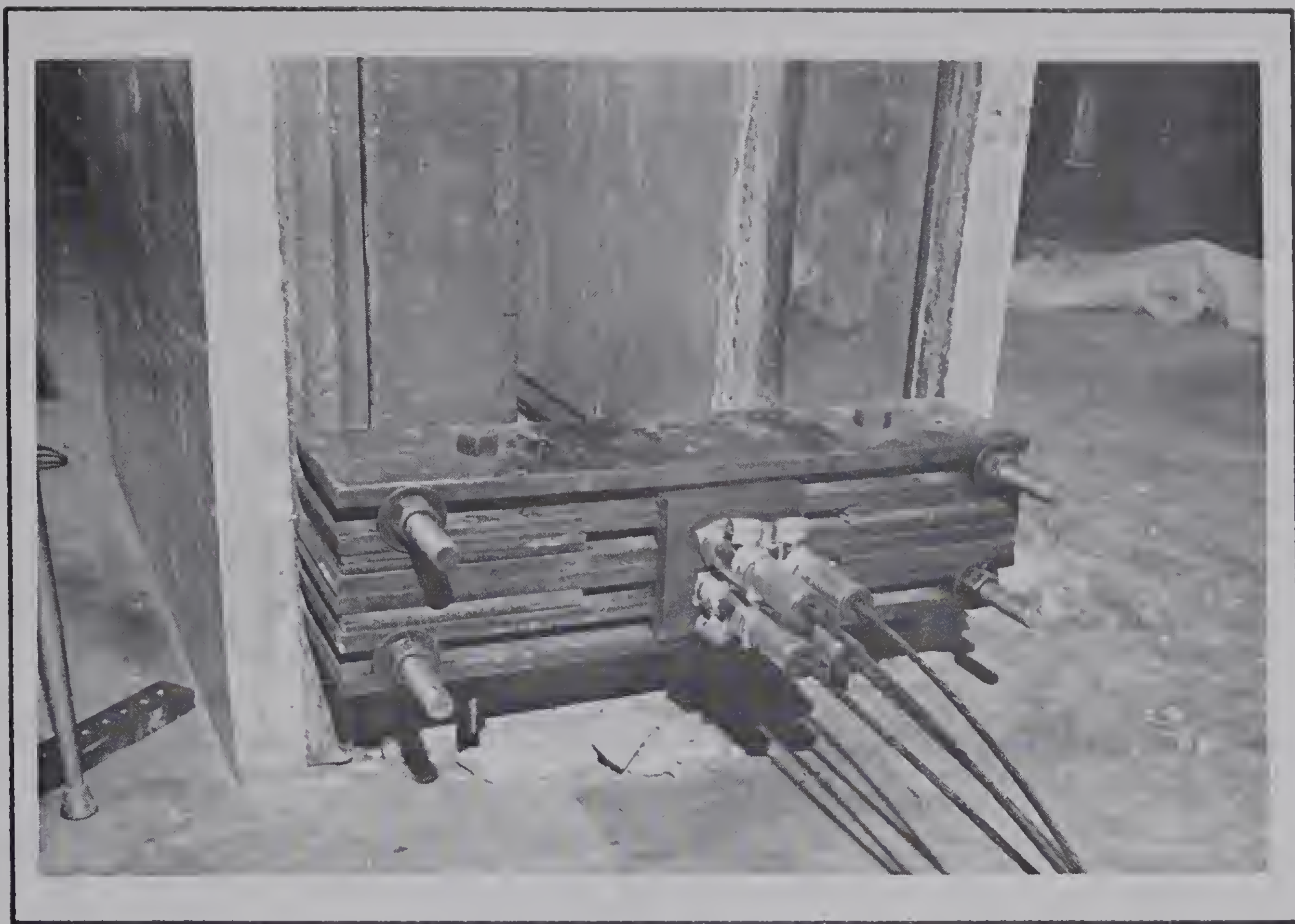


FIGURE 3.5 NORTH BULKHEAD

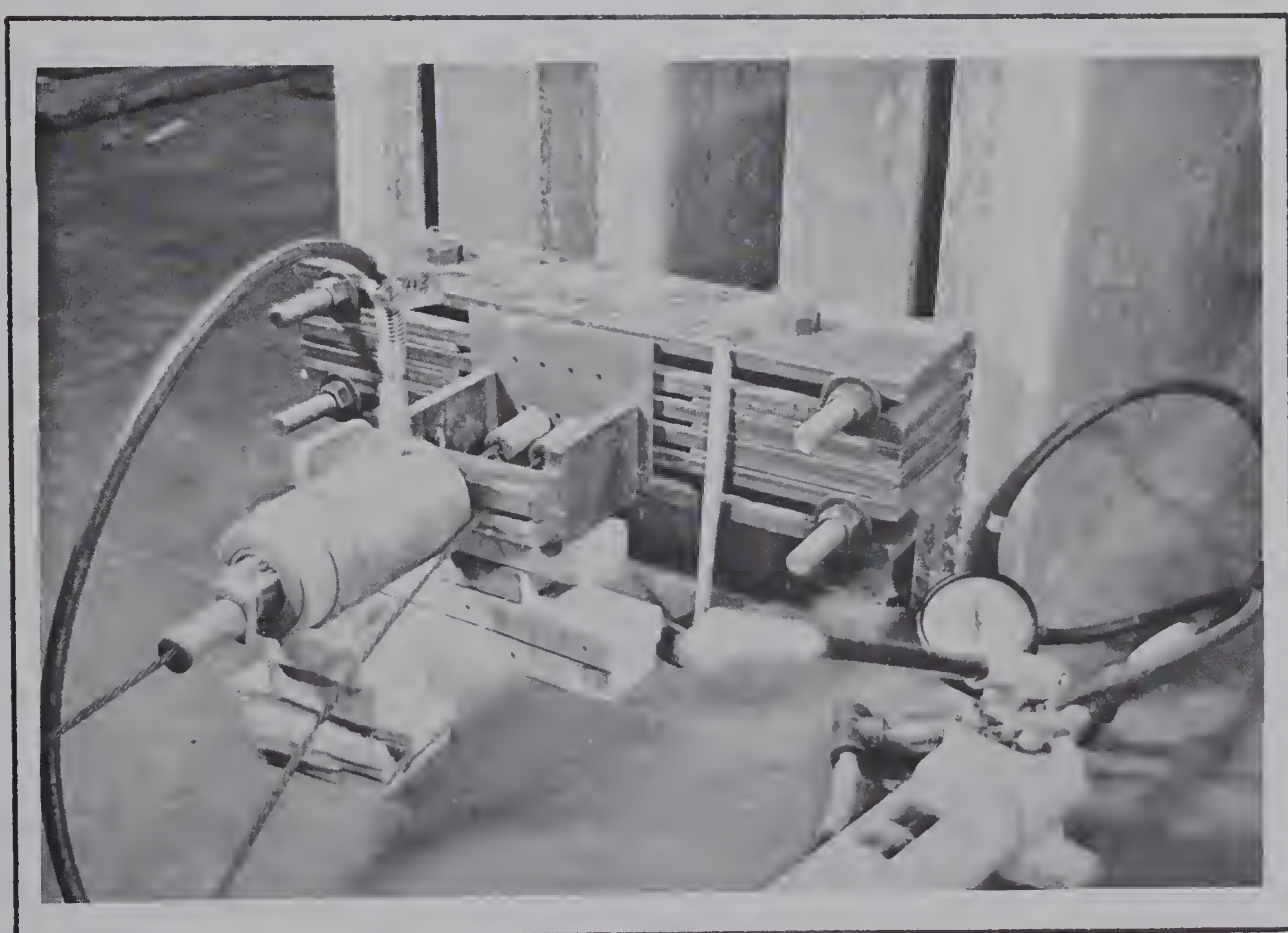
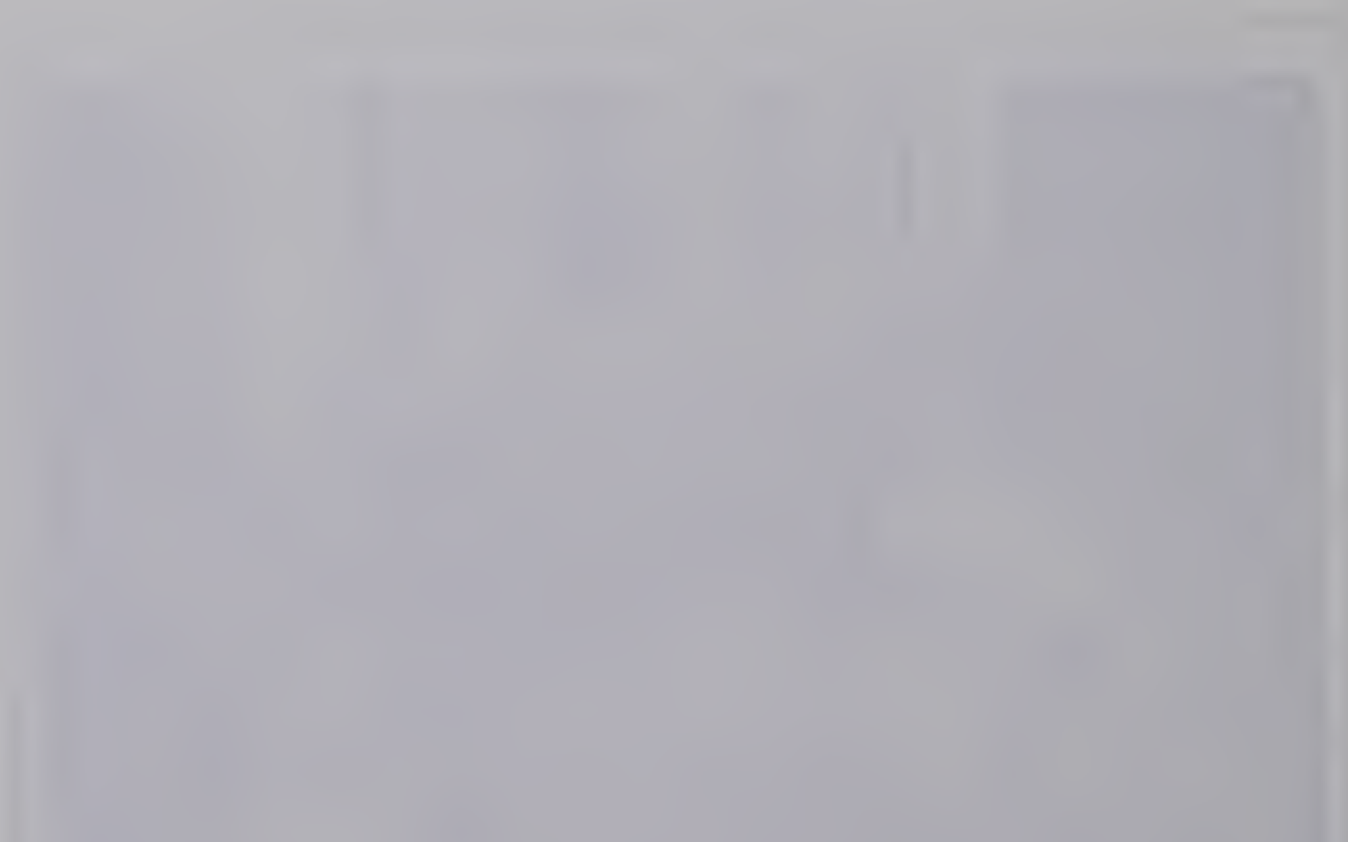


FIGURE 3.6 SOUTH BULKHEAD
(JACKING SYSTEM)



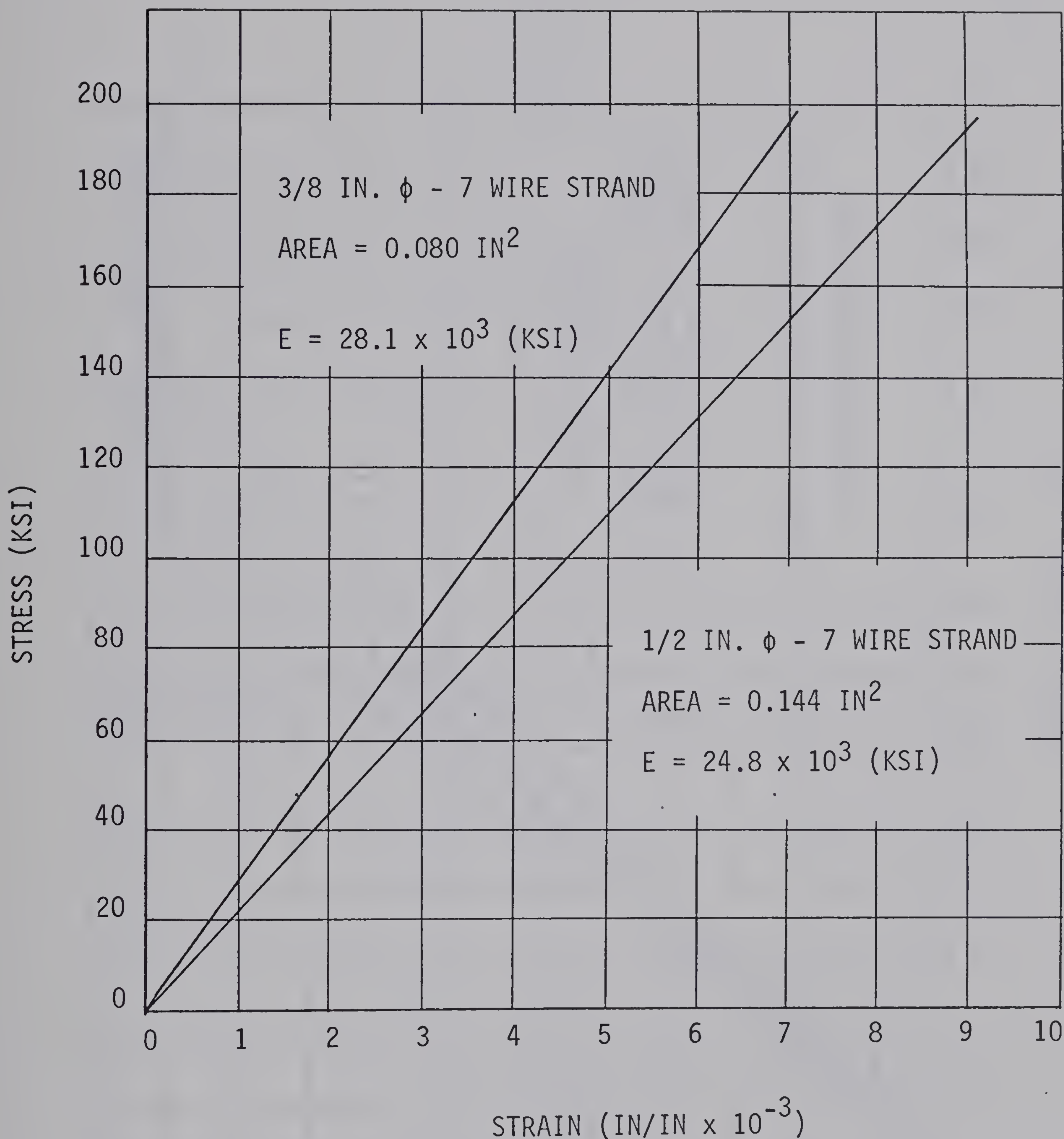


FIGURE 3.7 STRESS - STRAIN CURVES FOR PRESTRESSING STRAND

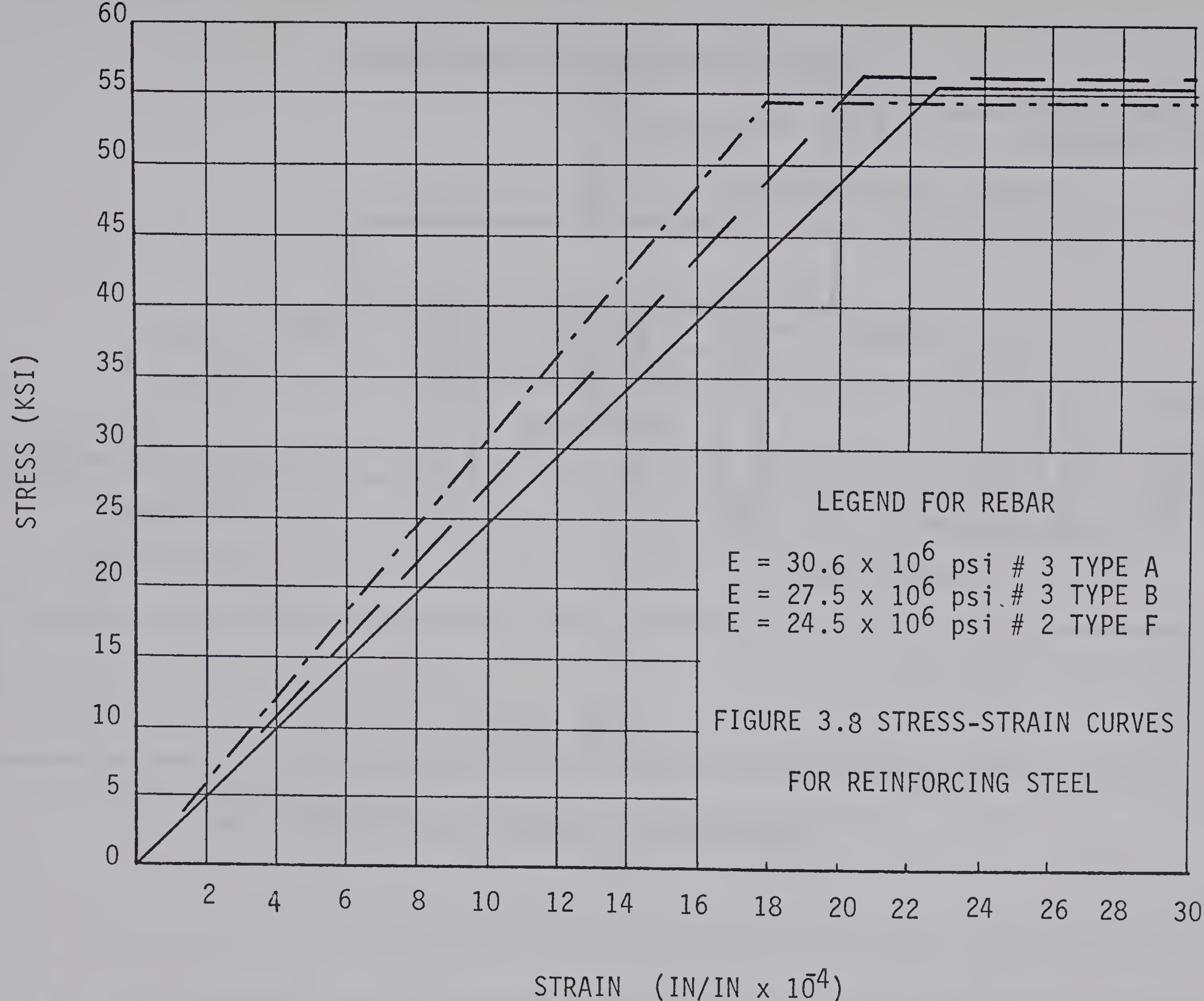


FIGURE 3.8 STRESS-STRAIN CURVES FOR REINFORCING STEEL

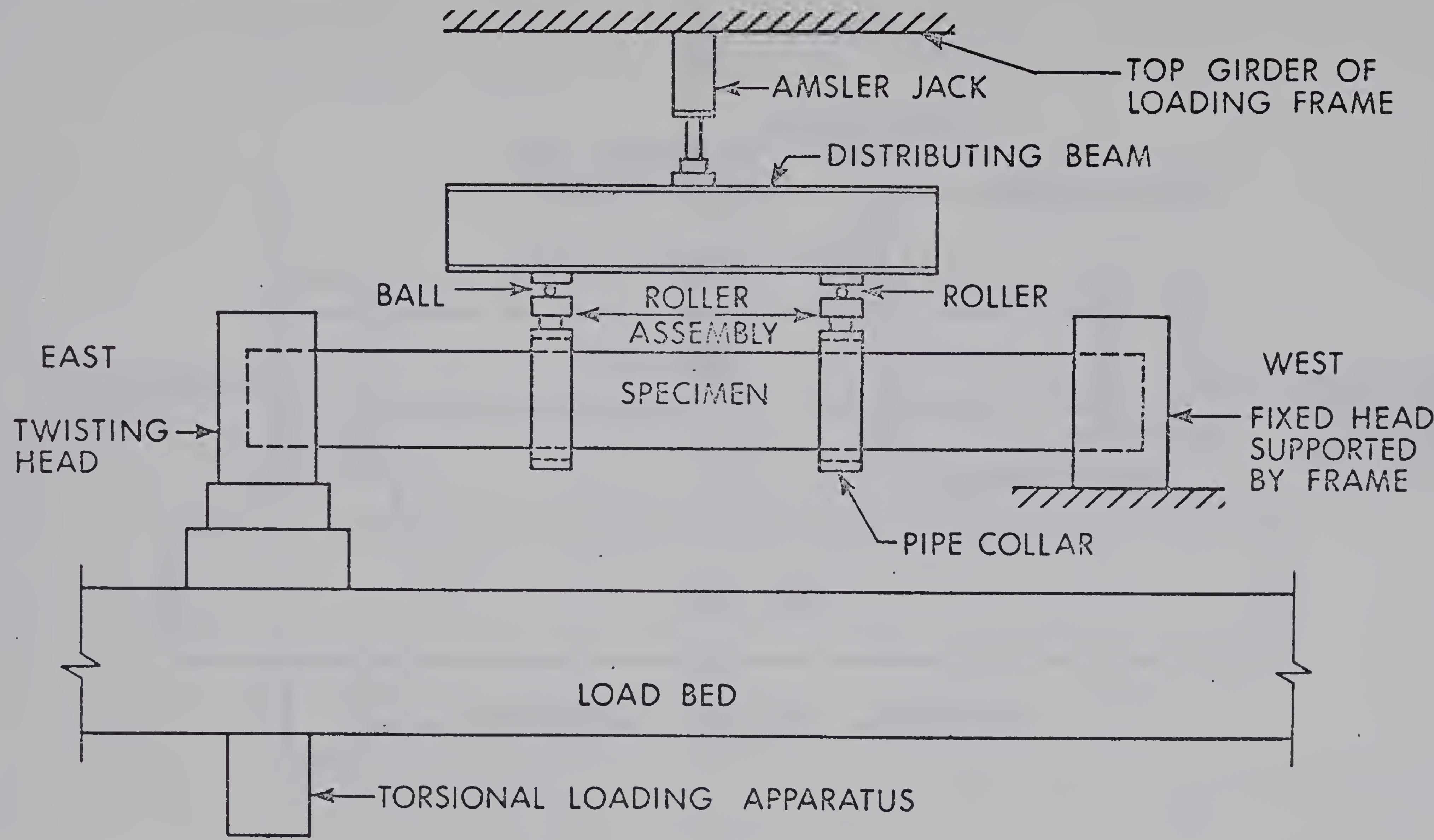


FIG. 3.9 EQUIPMENT ARRANGEMENT FOR COMBINED BENDING AND TORSION.

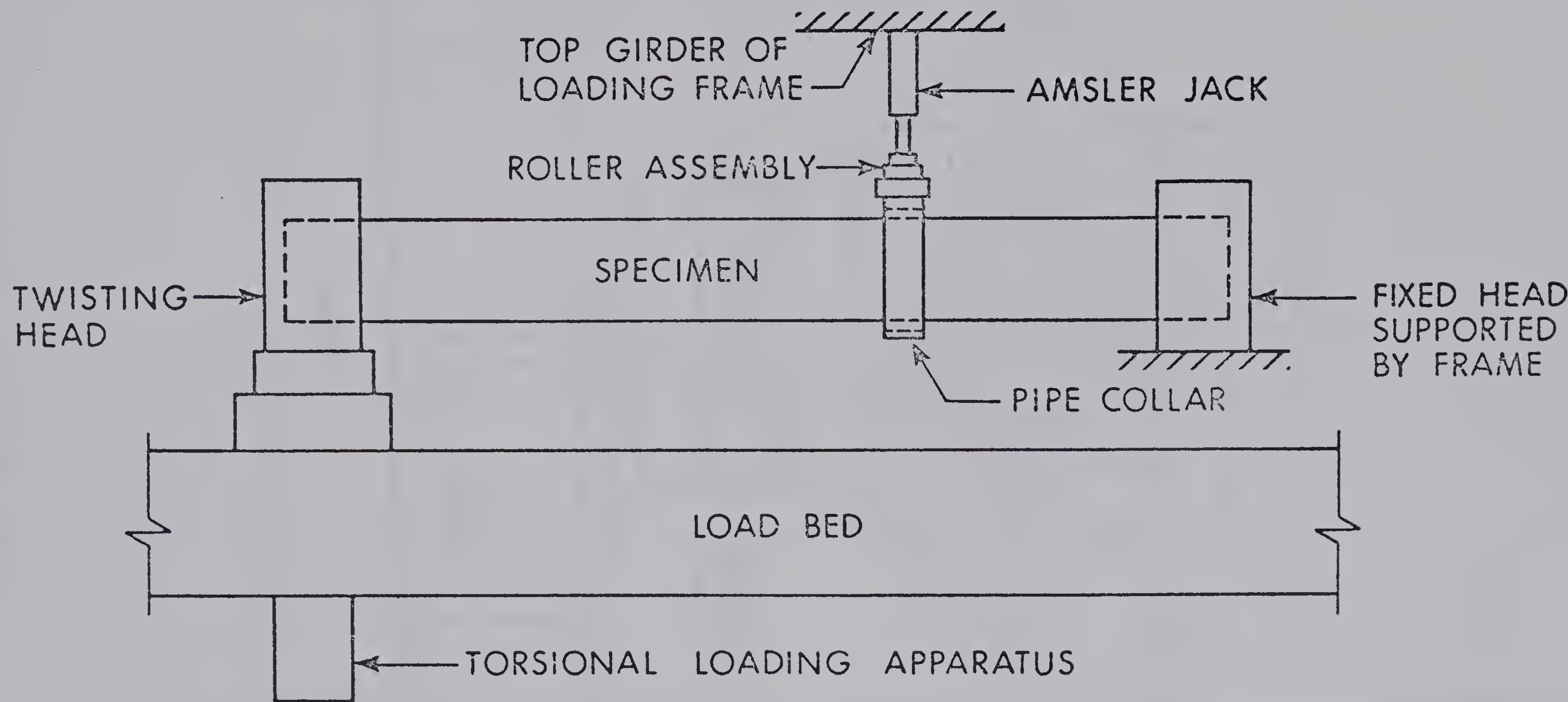


FIG. 3.10 EQUIPMENT ARRANGEMENT FOR COMBINED BENDING, TORSION AND SHEAR.

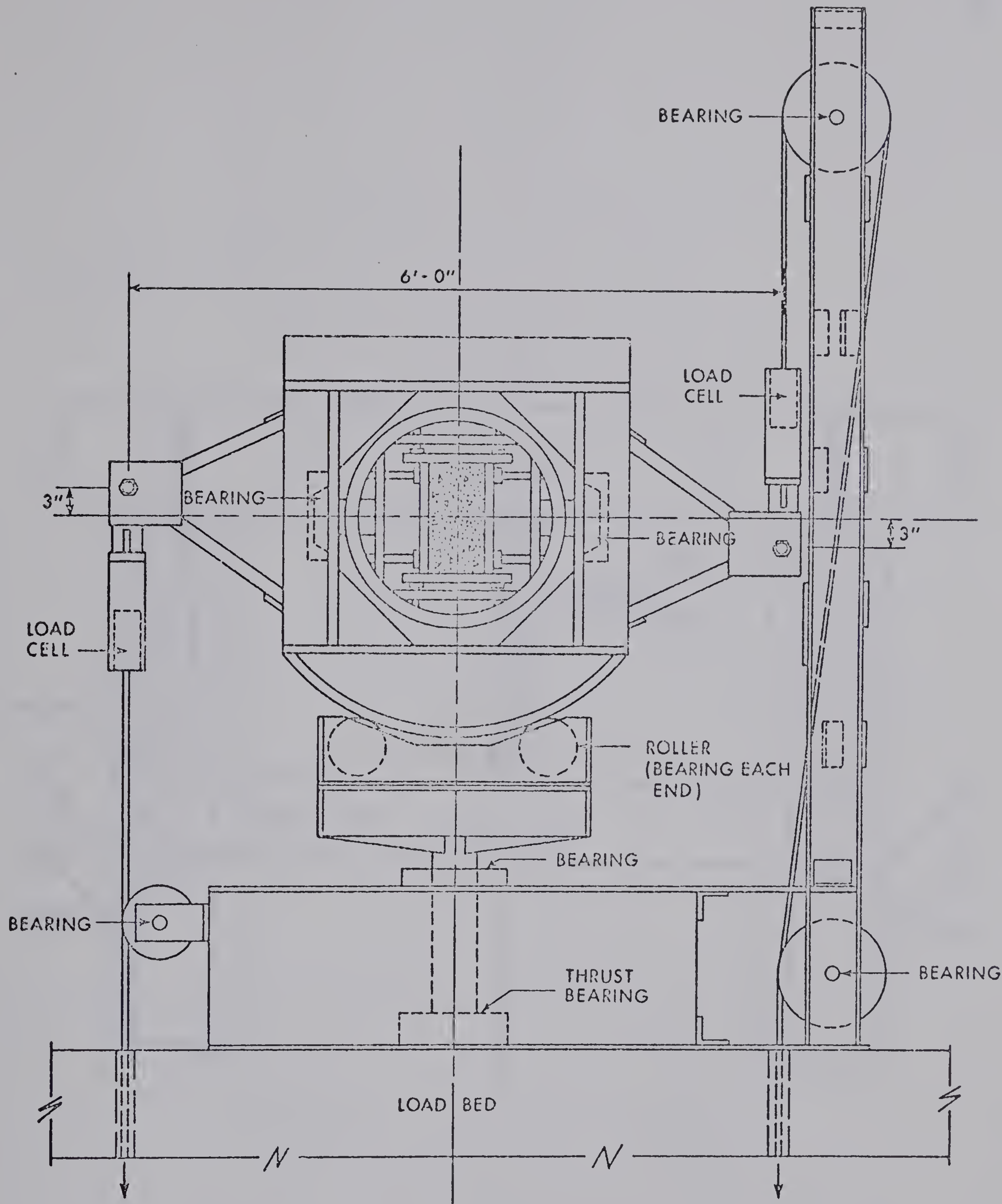


FIGURE 3.11 TWISTING HEAD

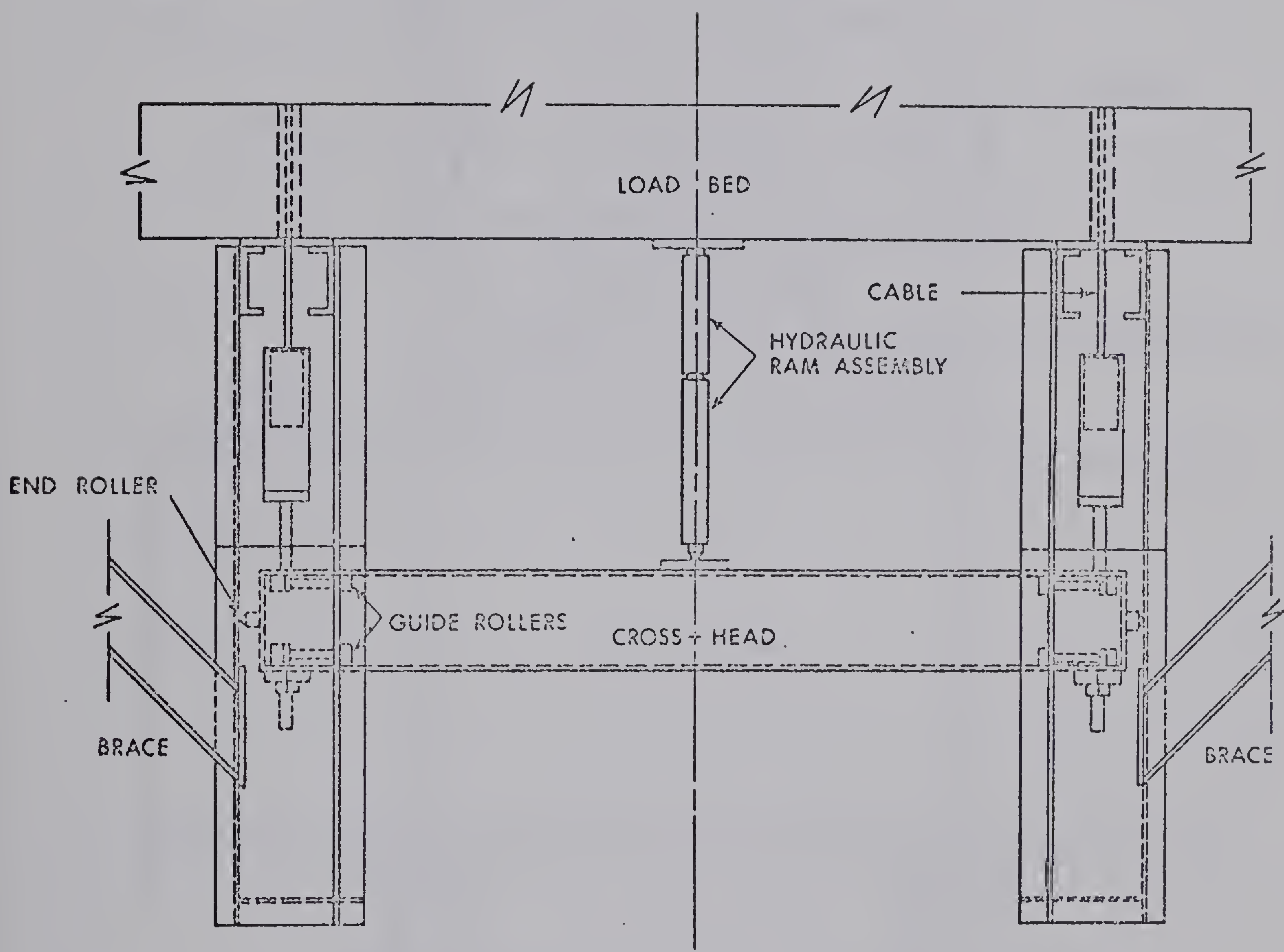


FIGURE 3.12 TORSIONAL LOADING SYSTEM

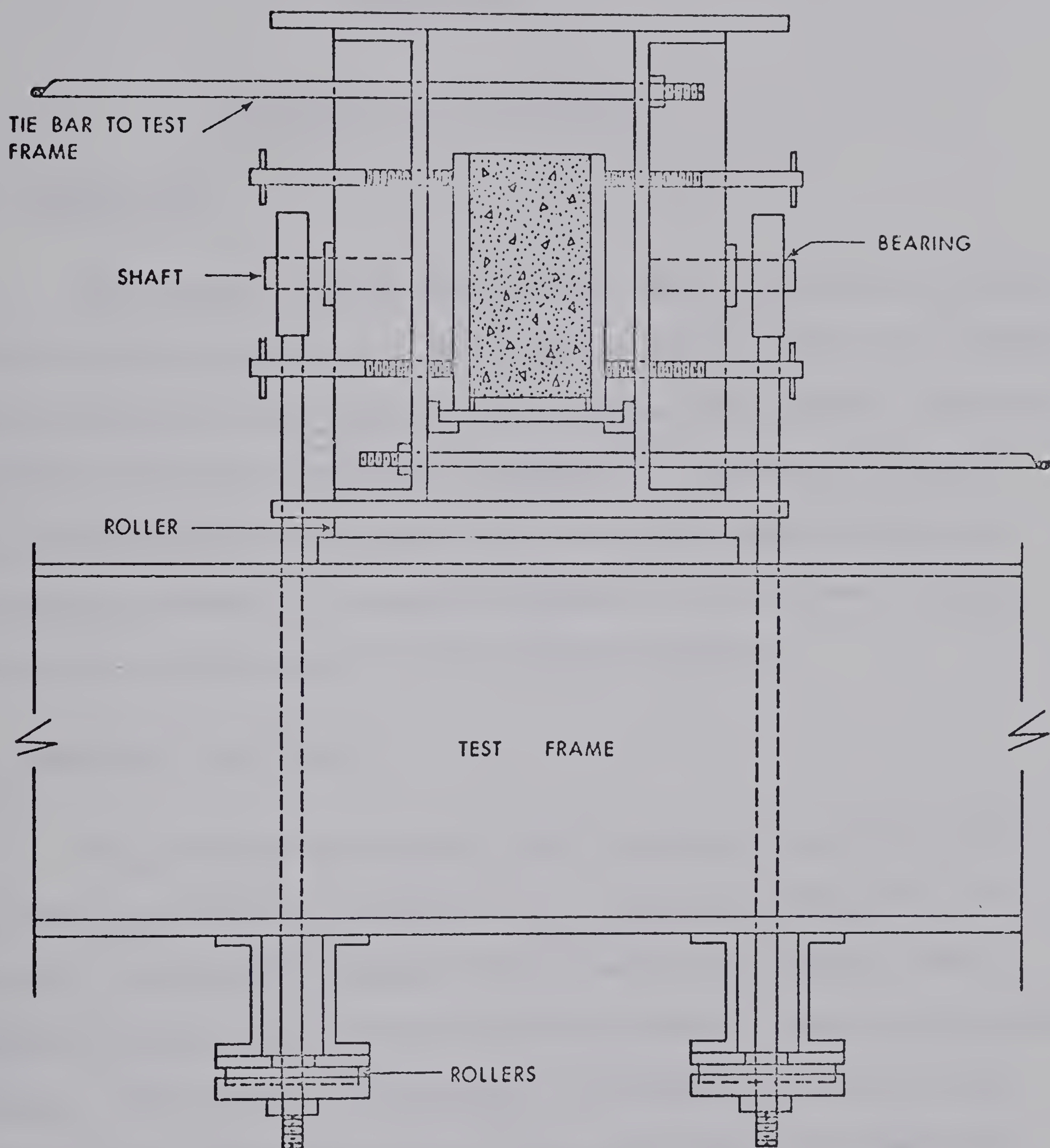


FIGURE 3.13 FIXED HEAD

CHAPTER IV

PRESENTATION OF TEST RESULTS

4-1 INTRODUCTION

This chapter presents the principal test results and the manner in which these were obtained. More detailed results such as the readings taken at the end of each load increment, torque-twist curves, and moment-deflection curves are presented in Appendix A. Interaction diagrams and plots of torsion to bending ratios versus the angle of twist are presented in Chapter V. In addition Chapter V also includes illustrations of the crack patterns for all the beams tested.

4-2 PRINCIPAL TEST RESULTS

The principal test results, which include the maximum moments and forces sustained by the members, are presented in Tables 4-1, and 4-2. The value listed as the maximum angle of twist for a specific beam is generally the twist recorded for the load increment immediately preceding failure. The ultimate bending moment is the moment existing at that section of the gauge length where failure took place. For beams subjected to high ratios of torsion to bending, the exact location of this failure plane was at times difficult to determine.

SERIES	BEAM NO.	$\frac{T}{M}$	APPLIED TORQUE AND MOMENT IN-KIPS T_c	IN-KIPS M_c	T_u	M_u	STEEL STRAIN AT ULTIMATE MICRO INCHES PER INCH GAUGE 1	2	3	4	θ_u $1x10^{-6}$ RAD/IN.	Δu INCH
I	301	0	0	445.5	0	796.5	129	-100	-153	663	0	0.99
	302	1/3	99.0	297.0	159.3	477.9	2200	1767	1122	348	746	0.73
	303	3/4	111.4	148.5	154.9	206.3	1241	1140	774	1204	970	0.17
	304	4/3	126.0	94.5	156.6	117.5	1428	2250	7218	946	1167	0.04
	305	3	129.6	43.2	164.5	54.7	583	-	2090	-	817	0.01
	306	∞	133.0	0	177.9	0	1094	633	5492	1483	632	
	307	1/7	55.9	391.5	94.5	661.5	472	288	75	160	349	0.85
II	321	0	0	486.0	0	904.5	-	-	-113	-	0	0.98
	322	1/3	99.0	297.0	154.2	465.8	1095	-	1094	197	1330	0.46
	323	3/4	121.5	162.0	172.1	229.5	1226	3318	4546	1247	1160	0.10
	324	4/3	126.0	94.5	162.0	121.5	296	1268	1420	641	624	0.03
	325	3	125.6	41.9	172.1	57.4	2217	2235	2610	1071	1395	-0.12
	326	∞	143.7	0	154.2	0	1173	3150	7977	-	932	
	327	1/7	67.5	472.5	123.5	864.0	483	284	766	257	123	1.26

TABLE 4.1 PRINCIPAL TEST RESULTS (GROUPS I & II)

SERIES	BEAM NO.	$\frac{T}{M}$ *	APPLIED TORQUE, MOM., SHEAR IN-KIPS		KIPS	D_f INCH +	STEEL STRAIN AT ULTIMATE MICRO INCHES PER INCH GAUGE				θ_u 1x10^{-6} RAD/IN	Δ_u INCH
			T_u	M_u	V_u		1	2	3	4		
III	V301	0	0	746.9	11.67	64	-	-32	-8	-141	0	0.53
	V302	1/3	119.6	542.8	9.20	59	-	-	1450	828	536	0.41
	V303	3/4	146.2	125.0	5.00	25	1900	1729	-	162	633	0.15
	V304	4/3	156.0	48.0	3.00	16	7203	-	1194	954	640	0.08
	V305	3	148.2	27.9	1.27	22	5018	1485	277	-	469	0.03
	V307	1/7	65.0	711.9	11.67	61	33	-28	-58	271	135	0.51
IV	V302P	1/3	104.0	368.0	8.00	46					208	0.25
	V322P	1/3	117.0	504.0	9.00	56					261	0.26

* Torque:Bending Moment Ratio at the centre of gauge length.
+ D_f = distance from the east end support to the failure zone.

TABLE 4.2 PRINCIPAL TEST RESULTS (GROUPS III & IV)

4-3 MOMENT - DEFLECTION RELATIONSHIPS

As mentioned in Section 3-4.2, six deflection gauges were used in order to determine the deflections of each specimen. The location of these gauges is shown in Figures 3.1, 3.2, and 3.3. Following the application of each load increment, readings were taken on the three sets of gauges with the average reading giving the deflection along the centerline of the specimen.

The bending moment for the beams subjected to bending and torsion only was calculated as the product of one half the total transverse load and the distance from one load point to the nearest support. For those beams which in addition were subjected to shearing forces, the maximum flexural moment occurred under the transverse load. For each load increment this moment was calculated as one-third the value of the transverse load multiplied by 72 inches, the result being in units of in-kips.

Knowing the bending moment and the deflections, the Moment-Deflection diagrams were plotted. In order to ensure legibility, only those points necessary to indicate the behavior of each beam were included.

4-4 TORQUE - TWIST RELATIONSHIPS

The two rotation gauges were mounted at the locations shown in Figures 3.1, 3.2, and 3.3. By subtracting the west gauge readings from those of the east gauge, the angle change between them was obtained. This angle change was then divided by the total length between the two gauges, yielding the angle of twist of the beams in units of radians

per inch.

From a calibration chart of the load cell connected in series with the twisting head cables, it was possible to obtain the force existing in those cables which corresponded to a particular load cell reading. The torsional moment acting upon the specimen was then evaluated as the force in the twisting head cables multiplied by the moment arm of 72 inches.

The torsional characteristics of the specimens are presented in the form of Torque-Twist curves. These curves include only those points necessary to completely indicate the behavior of each specimen.

4-5 EFFECTIVE PRESTRESS FORCE

In order to allow a correlation to be made between the different test results, it was necessary to obtain the value of the effective prestress force acting on each specimen. This was accomplished by taking measurements determining the elastic shortening and the time dependent losses as discussed previously in Section 3.5. Converting the strain losses in the prestressed reinforcement into resulting stresses, the total loss in strand force from the time of release until the time of testing could be computed. Deducting these losses from the strand forces prior to release resulted in the effective prestress force at the time of testing. Although slight variations in the effective prestress force and concrete strength between the specimens occurred, it was felt that the ratios obtained were quite acceptable.

4-6 INTERACTION DIAGRAMS

The effect of the simultaneous application of different types of forces on a beam can be conveniently expressed by means of an interaction diagram. In this way the interaction of torsion and bending can be represented on a two-dimensional, rectangular coordinate system. However, combinations of three different types of forces such as bending, torsion, and shear would have to be shown on a three-dimensional interaction surface.

In this study all the interaction diagrams have been presented as two-dimensional plots. For the beams of Groups I and II the curves were of dimensional and non-dimensional form for both cracking and ultimate loads. These diagrams are illustrated in Figures 5.1A, 5.1B, 5.2A, and 5.2B. For the beams of Group III which were subjected to a combined loading of bending, torsion, and shear, interaction diagrams were plotted at ultimate with the transverse shear and flexural moment respectively plotted versus the torsional moment. The cracking torque of a member was generally found as the torque existing in that member at a time when a noticeable increase occurred in the strain gauge readings for the transverse reinforcement in the gauge length. Similarly, the ultimate torsional moment was that value of torque which existed on the specimen at the time of failure. Due to variations in the location of the failure plane for the beams tested in combined loadings of bending, torsion, and shear, the ultimate bending moment was considered to be the moment existing at the plane of failure when failure occurred. As mentioned in Section 4-2, the location of the failure plane was at times

difficult to determine.

The value of the shear along the gauge length, V_u, was taken to be one-third the value of the applied transverse load at the time of failure. In Table 5.1 the value used for V_{uo} as one interaction parameter was taken to be the shear existing in the gauge length at the time of failure for a beam subjected to bending and shear only. However, in order to make a valid comparison between these experimental findings and the empirical equations derived by Mukherjee and Warwaruk(2), the pure shear strength, V_{uo}, of the beams as used in Table 5.2 was computed on the basis of the provisions in ACI 318-63 (Equations 26-10, and 26-13). The interaction values I_c and I_u in Table 5.2 were determined by evaluating the empirical equations with the appropriate test parameters. For an exact correlation, the interaction values should be 1.0 in all cases.

For the specimens subjected to bending, torsion, and shear, the dimensional and non-dimensional interaction diagrams were plotted in Figures 5.3A and 5.3B.

4-7 REINFORCEMENT STRAINS

The reinforcement strains were measured only on the transverse non-prestressed reinforcement. These strains are shown for the complete loading history in tabular form in Appendix A, and only at ultimate in Table 4.1 and Table 4.2. No attempt was made to measure the strains in the prestressing reinforcement as no practical method for doing this was available. Knowing the level of stress in the transverse reinforcement was helpful in interpreting and understanding the interaction between the steel and the concrete at various stages of loading.

CHAPTER V

DISCUSSION OF TEST RESULTS

5-1 INTRODUCTION

The purpose of this chapter is to study the behavior of the test specimens at various stages of loading in the loading sequence. The effect of such test parameters as degree of prestress, eccentricity, torque to bending ratio, and stirrup spacing is discussed. In addition, information dealing with the interaction of bending, torsion, and shear is presented and summarized in interaction tables and diagrams.

5-2 GENERAL BEHAVIOR

Each of the beams tested in this series exhibited essentially two stages of behavior over the loading sequence. In the first stage the beams behaved elastically with little or no change in the strain gauge readings for the non-prestressed transverse reinforcement. The end of this stage was marked by extensive cracking of the members.

For the beams which contained transverse reinforcement a second stage was evident following cracking marked by a continued increase in strength of the specimen. This stage was further characterized by a large decrease in the torsional stiffness of the beam, and the load deformation characteristics were no longer linear.

Following cracking, those beams which contained no transverse

reinforcement could no longer sustain any increases in the applied torsional moment. The resultant failure in the case of the concentric specimen was abrupt and quite violent, whereas the failure of the eccentric beam was less forceful.

5-3 BEHAVIOR UNDER COMBINED BENDING AND TORSION

The crack patterns associated with beams from Groups I and II are illustrated in Figures 5.7 and 5.8. The location and formation of these cracks depended primarily on the torque to moment ratio to which the beams were subjected.

In the case of bending with little or no torsion, the initial crack formation occurred on the bottom of the specimens and continued vertically upwards, defining the lower boundary of the compression zone. Immediately inside the compression zone the cracks propagated at a sharp inclination towards the point of load application, generally stopping some distance from the top of the beam. Failure occurred quite close to the load point and was evidenced by visible crushing of the top fibres.

In the medium range of torque to bending, the cracks originated almost simultaneously on the bottom and side faces of the members. Near the bottom the cracks were essentially vertical, becoming progressively more inclined to the vertical axis as they propagated upwards through the beams. Failure by crushing of the concrete took place along one of these inclined cracks usually at some distance from the load point. As the proportion of torsional moment was increased still further, the crack formation initiated near the center of the

vertical faces, often in the area close to the center of the gauge length. These cracks progressed towards the top and bottom faces along lines which were essentially straight and oriented to the horizontal axis of the members at an inclination of about 30°. A particularly wide crack with evidence of crushing in the concrete indicated the failure plane of these beams. In most cases the crack which widened at failure and defined the failure plane had first appeared at a bending moment which was considerably lower than the ultimate moment. These cracks were all essentially a continuous crack which spiralled around three faces of the beam. Such a crack is illustrated in Figure 5.6 which shows the failure surface of Beam V302P.

Table 4.1 lists the level of strain in the non-prestressed transverse reinforcement at ultimate. As presented, the reinforcement did not yield at ultimate when the applied torsion to bending ratio was 1:3 or lower, except in the case of Beam 302. In all other cases the transverse steel had yielded, extending into the strain hardening region. The non-prestressed transverse reinforcement in addition to the horizontal prestressed reinforcement therefore served as a means to increase the strength of the beams over and above that at cracking, as well as to impart considerable post-cracking ductility. To a large extent then, the post-cracking behavior of a beam is determined by the amount and type of reinforcement provided for that member.

A comparison of the torque-twist curves in Appendix A to torque-twist curves of similar beams plotted by Mukherjee and Warwaruk (3) reveals that the slopes are lower in the post-cracking region than those of Mukherjee. The non-prestressed transverse reinforcement used

in both studies is the same and only the spacing is changed. In addition Mukherjee used non-prestressed longitudinal reinforcement located in each corner of the beams. In the case of the prestressed reinforcement, in this study six prestressing cables are used as opposed to four by Mukherjee, four of which are located in each corner of the beams. The effect of this, if anything, would be to increase the torsional stiffness of the beams due to the increase in lever arm of the prestressing cables. It would thus seem a justifiable conclusion that the torsional stiffness of a beam is decreased in the post-cracking region with an increase in the spacing of the transverse reinforcement. In the pre-cracking stage the comparison indicates that the torsional stiffness is independent of both the amount and location of the prestressing reinforcement as well as the amount and spacing of the non-prestressed reinforcement.

From Table 4.1 it becomes apparent that beams prestressed concentrically generally exhibited less ductility than beams which were prestressed eccentrically. Furthermore, the ultimate torsional strength in general was not adversely affected by the eccentricity of prestress, and in fact a small increase was often noted. Hence the location of the prestressing strands is an added factor which may affect the rotational capacity of a beam.

Another interesting observation which can be made by comparing the experimental results to those obtained by Mukherjee and Warwaruk (3) is the effect of the level of prestress on the strength of a beam. With the exception of Beams 304 and 322, in every case the effect of increasing the level of prestress was to increase the capacity at ul-

timate of the specimens. This increase in strength, however, was achieved at the expense of a loss in ductility. The amount of post-cracking behavior was reduced causing the beams to fail in a brittle manner with only a small amount of ductility exhibited in the post-cracking region. It would thus be desirable to limit the allowable level of prestress in a beam in order to ensure that adequate warning of an impending failure would be given. In the pre-cracking region the effect of increasing the level of prestress was to delay the formation of the initial cracks in the members, thus causing an increase in the cracking torques of the beams.

Another variable which may affect the ultimate ductility of a beam is the torsion to bending ratio. This dependency is illustrated in Figure 5.5 where the ultimate angle of twist is plotted versus the torque to bending ratio. Although a definite relationship seems to exist for the beams of Groups I and III, the points applying to Group II beams were too scattered to present any form of relationship. This may be due to the seemingly greater sensitivity to variations in the applied loading which Group II beams exhibited in comparison to beams of Groups I and III.

5-4 INTERACTION BETWEEN TORSION AND BENDING

The main objective of the present investigation was to study the effect of bending and torsion on the capacity of a beam. To facilitate this, interaction diagrams have been plotted in Figures 5.1A, 5.1B, 5.2A, and 5.2B showing the effect of simultaneous applications of bending and torsion in varying ratios.

The pure torsional strength of Beam 306 was considerably higher than anticipated, perhaps due to the relatively high concrete strength of this specimen as compared to the other members of Group I. In plotting the interaction diagrams, because of uncertainty in the experimental strength of Beam 306, a value of T_{uo} was obtained by extrapolation of the dimensional interaction curve for the Group I beams in Figure 5.1B. The value obtained was about 160 In-Kips which corresponded very closely to the theoretical value of 159.3 In-Kips computed from the equation derived by Mukherjee (5) for concentrically prestressed beams containing web reinforcement. It is this value of 159.3 In-Kips which has been used in computing the interaction parameters listed in Tables 5.1 and 5.2.

Table 5.2 shows a comparison between the experimental findings of the present investigation and a interaction relationship which was derived empirically by Mukherjee and Warwaruk (3). The interaction values I_c and I_u should be 1.0 for an exact correlation between the theoretical and experimental values. For the concentrically prestressed beams of Group I the average value of I_u was 0.950 indicating a variation of only 5%. The agreement was even better for the eccentrically prestressed beams. As shown in Table 5.2 the variation in this case was only 1.3%, thus implying that the proposed interaction surface is quite compatible with experimental findings.

To illustrate further the correspondence between the obtained test data and the strength predictions, a plot of the computed theoretical values is included in Figure 5.1B. These predicted values were computed using various torsion to bending moment ratios in the equation

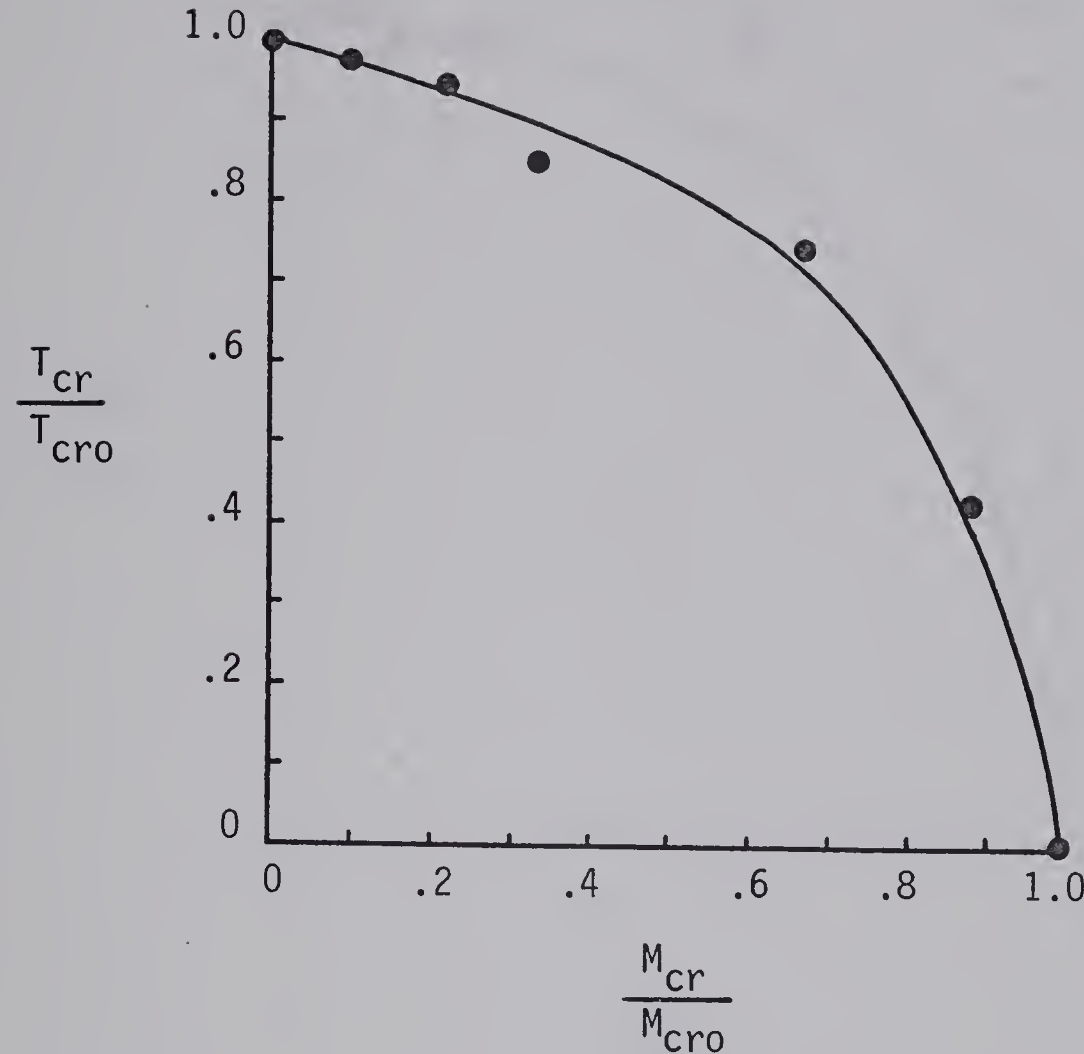
SERIES	BEAM NO.	CRACKING		ULTIMATE		
		$\frac{T_c}{T_{co}}$	$\frac{M_c}{M_{co}}$	$\frac{T_u}{T_{uo}}$	$\frac{M_u}{M_{uo}}$	$\frac{V_u}{V_{uo}}$
I	301	0	1.000	0	1.000	
	302	0.744	0.667	0.998	0.600	
	303	0.838	0.333	0.971	0.259	
	304	0.947	0.212	0.981	0.148	
	305	0.974	0.097	1.031	0.069	
	306	1.000	0	1.116	0	
	307	0.420	0.879	0.592	0.831	
II	321	0	1.000	0	1 000	
	322	0.689	0.611	1.000	0.515	
	323	0.846	0.333	1.116	0.254	
	324	0.877	0.194	1.051	0.134	
	325	0.874	0.086	1.116	0.063	
	326	1.000	0	1.000	0	
	327	0.470	0.972	0.801	0.955	
III	V301			0.000	0.937	1.000
	V302			0.750	0.681	0.788
	V303			0.916	0.157	0.428
	V304			0.978	0.060	0.257
	V305			0.928	0.035	0.109
	V307			0.407	0.893	1.000
IV	V302P			0.653	0.461	0.685
	V322P			0.759	0.633	0.771

TABLE 5.1 INTERACTION PARAMETERS

	BEAM NO.	$2\frac{\sigma}{f'_c}+\frac{e}{d}$	$\frac{T_c}{T_{co}}$	$\frac{M_c}{M_{co}}$	I_c	$\frac{T_u}{T_{uo}}$	$\frac{M_u}{M_{uo}}$	$\frac{V_u}{V_{uo}}$	$\frac{T_u/T_{uo}}{M_u/M_{uo}}$	I_u
I	301	0.552	0.000	1.000	1.000	0.000	1.000			1.000
	302	0.664	0.744	0.667	0.746	0.998	0.600			0.960
	303	0.706	0.838	0.333	0.714	0.971	0.259			0.855
	304	0.624	0.947	0.212	0.860	0.981	0.148			0.911
	305	0.600	0.974	0.097	0.925	1.031	0.069			0.995
	306	0.560	1.000	0.000	1.000	1.116	0.000			1.116
	307	0.562	0.420	0.879	0.879	0.592	0.831			0.817
AVERAGE					0.874					0.950
II	321	0.755	0.000	1.000	1.000	0.000	1.000			1.000
	322	0.743	0.689	0.611	0.611	1.000	0.515			0.882
	323	0.681	0.846	0.333	0.730	1.116	0.254			1.008
	324	0.815	0.877	0.194	0.757	1.051	0.134			0.960
	325	0.769	0.874	0.086	0.815	1.116	0.063			1.072
	326	0.761	1.000	0.000	1.000	1.000	0.000			1.000
	327	0.759	0.470	0.972	0.972	0.801	0.955			0.989
AVERAGE					0.840					0.987
III	V301	0.570				0.000	0.937	0.240	0.00	0.937
	V302	0.592				0.750	0.681	0.188	1.10	0.912
	V303	0.554				0.916	0.157	0.103	5.84	0.954
	V304	0.582				0.978	0.060	0.062	16.30	1.009
	V305	0.634				0.928	0.035	0.026	26.55	0.933
	V307	0.570				0.407	0.893	0.240	0.46	0.893
AVERAGE										0.940

I_u Computed on basis of T_{uo} being 159.3 IN.-KIPS

TABLE 5.2 COMPARISON BETWEEN EXPERIMENTAL RESULTS AND MUKHERJEE'S PROPOSED INTERACTION SURFACE



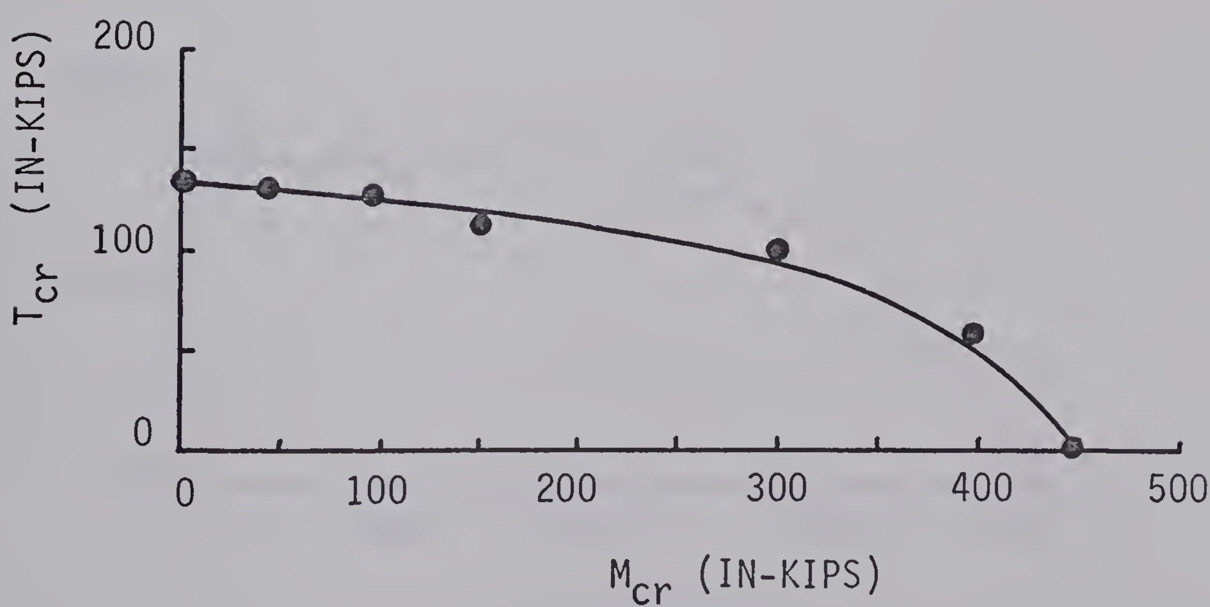


FIGURE 5.1A INTERACTION DIAGRAMS (GROUP I BEAMS)

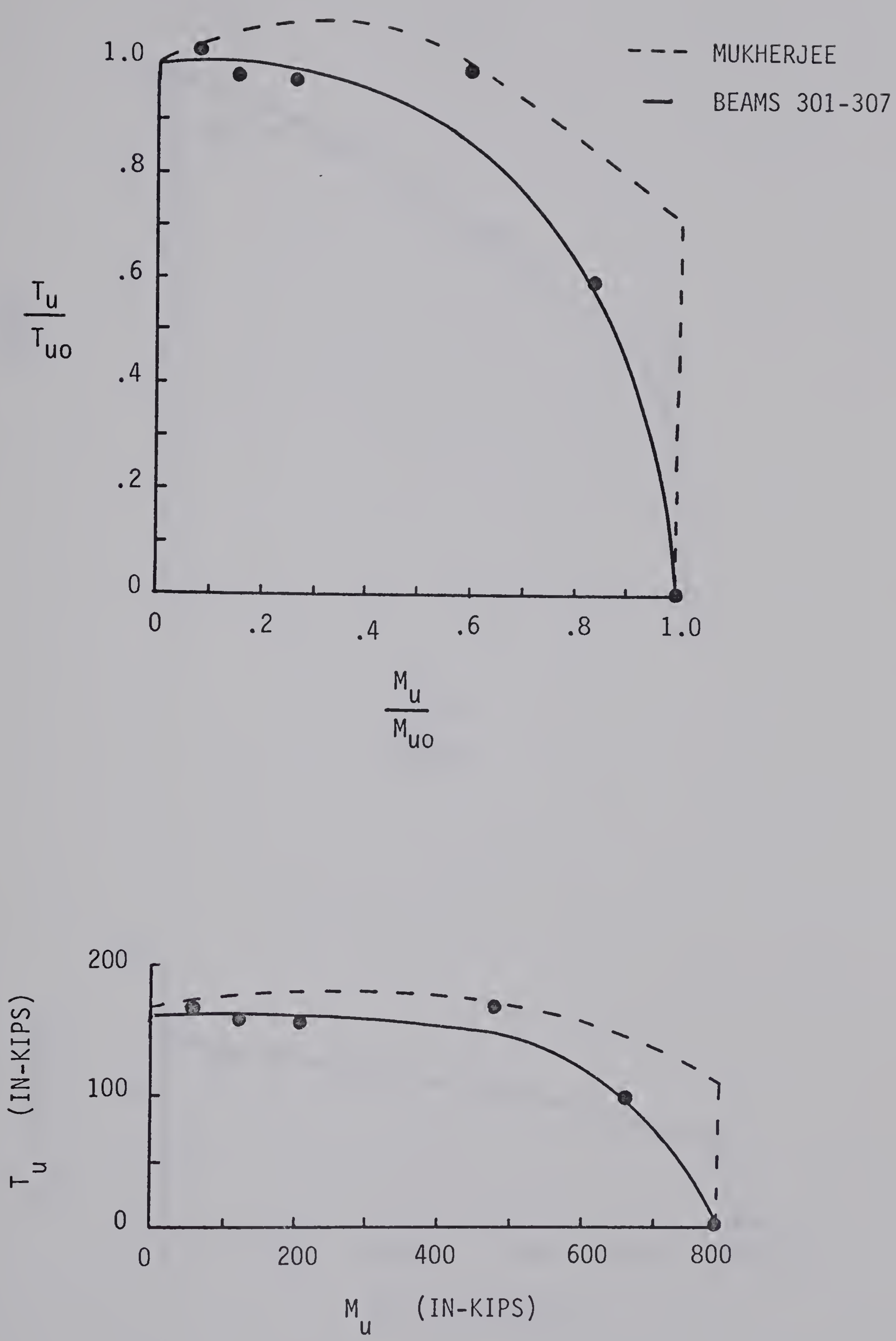
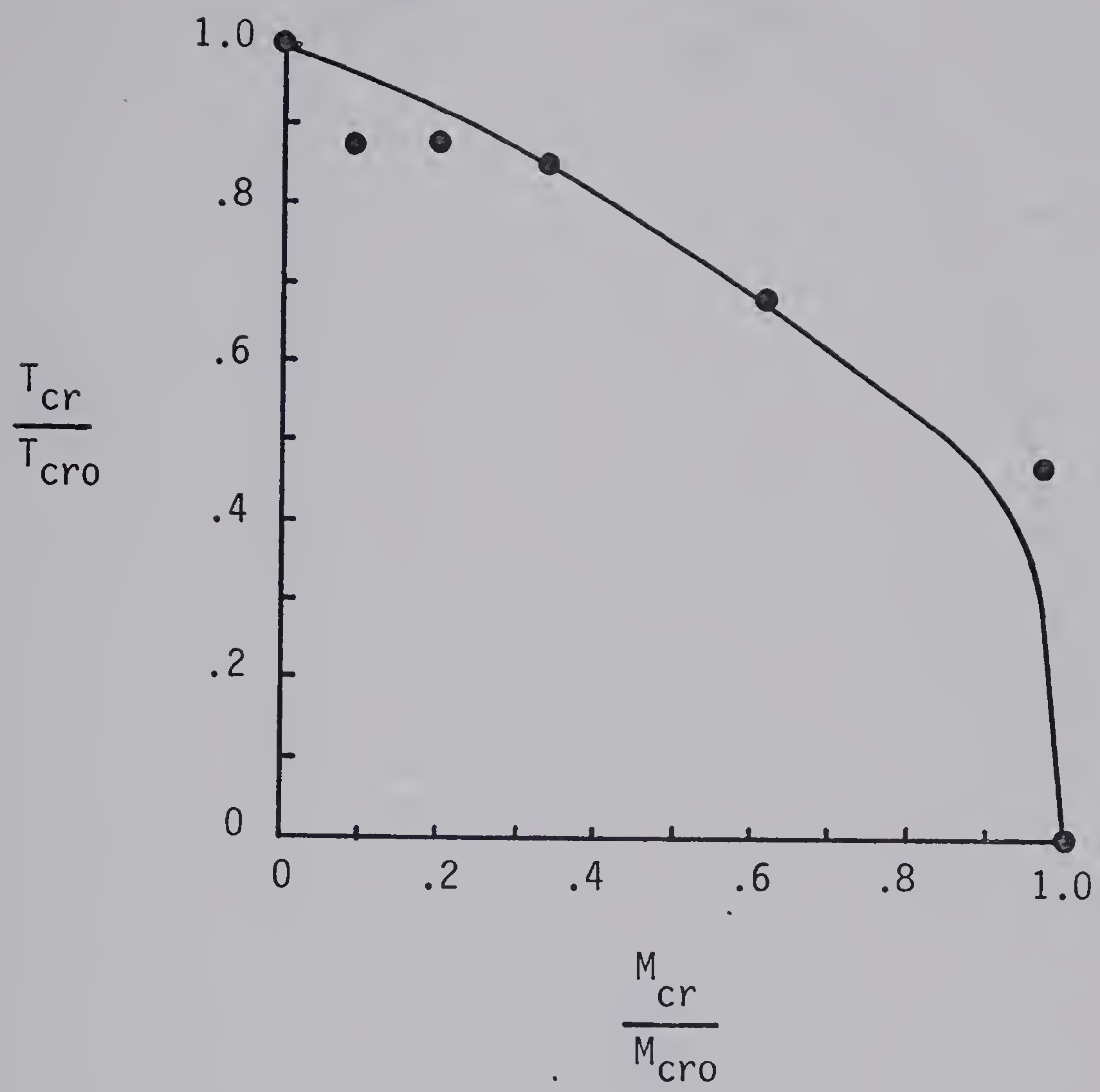


FIGURE 5.1B INTERACTION DIAGRAMS (GROUP I BEAMS)



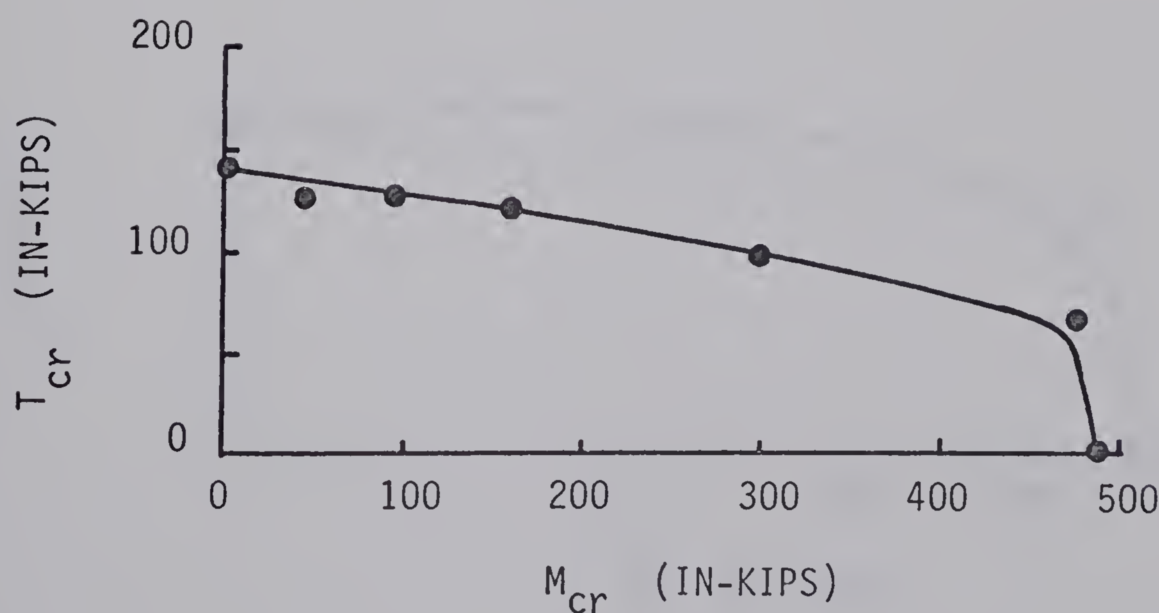


FIGURE 5.2A INTERACTION DIAGRAMS (GROUP II BEAMS)

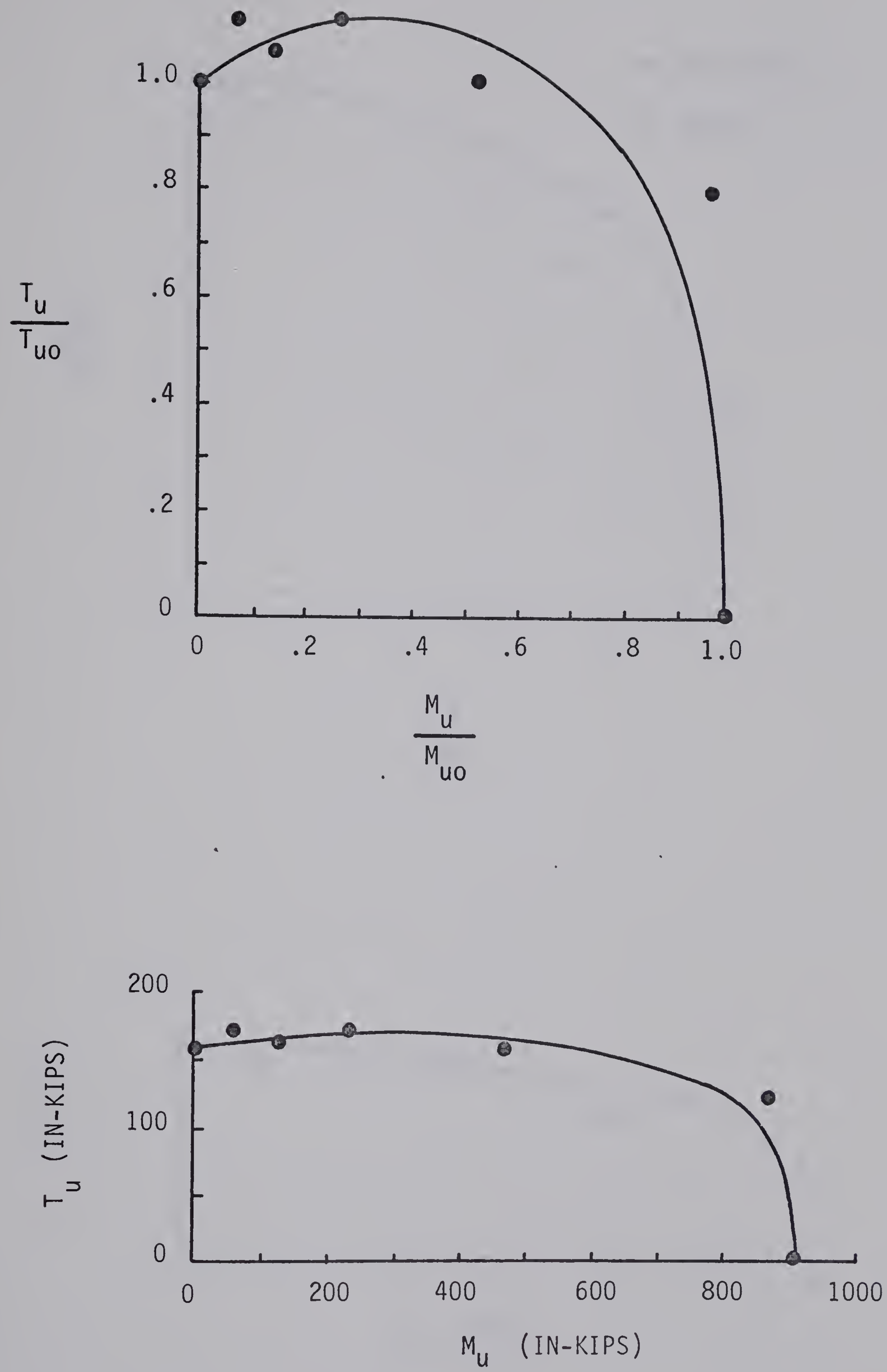


FIGURE 5.2B INTERACTION DIAGRAMS (GROUP II BEAMS)

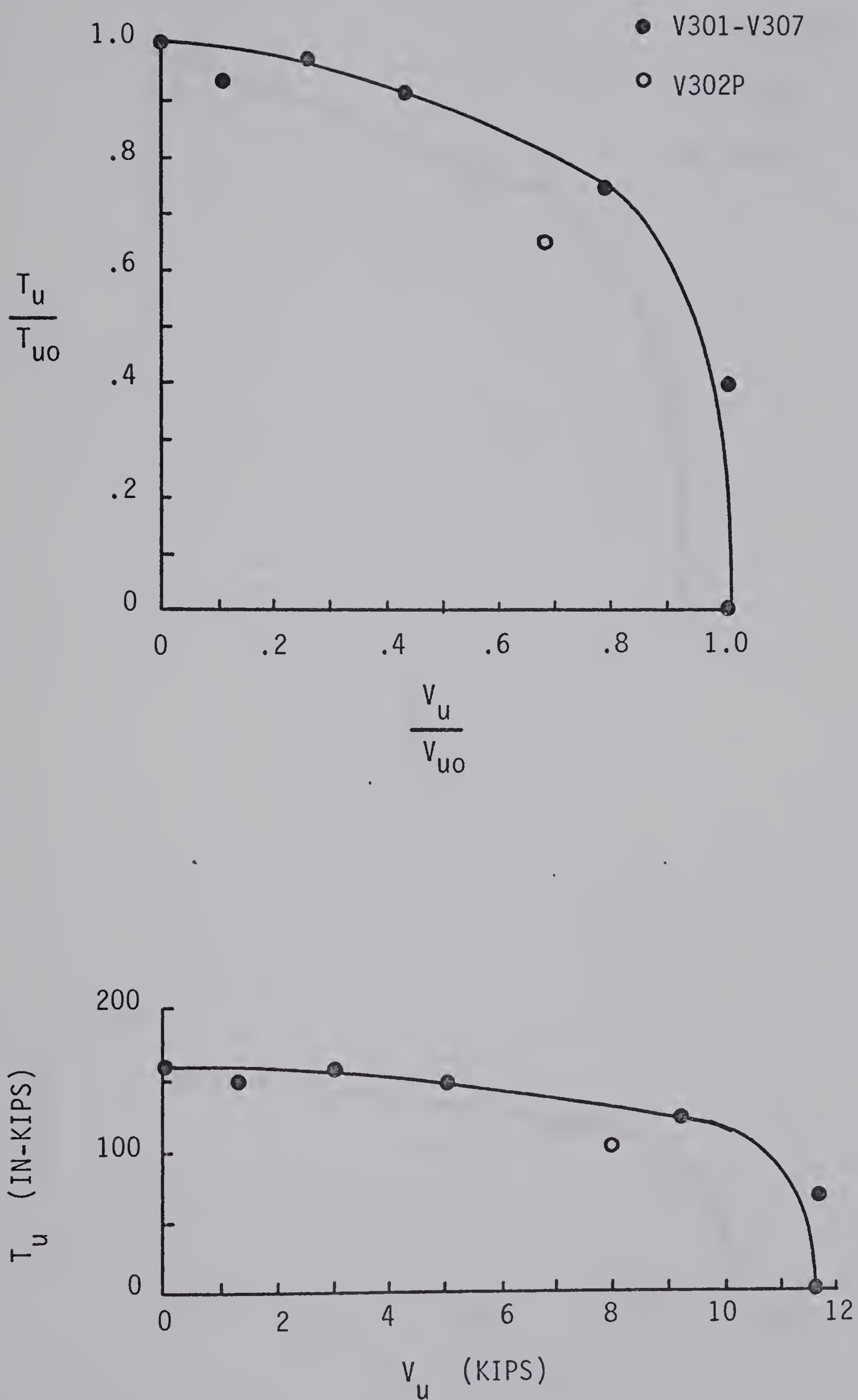


FIGURE 5.3A INTERACTION DIAGRAMS (GROUP III & IV BEAMS)

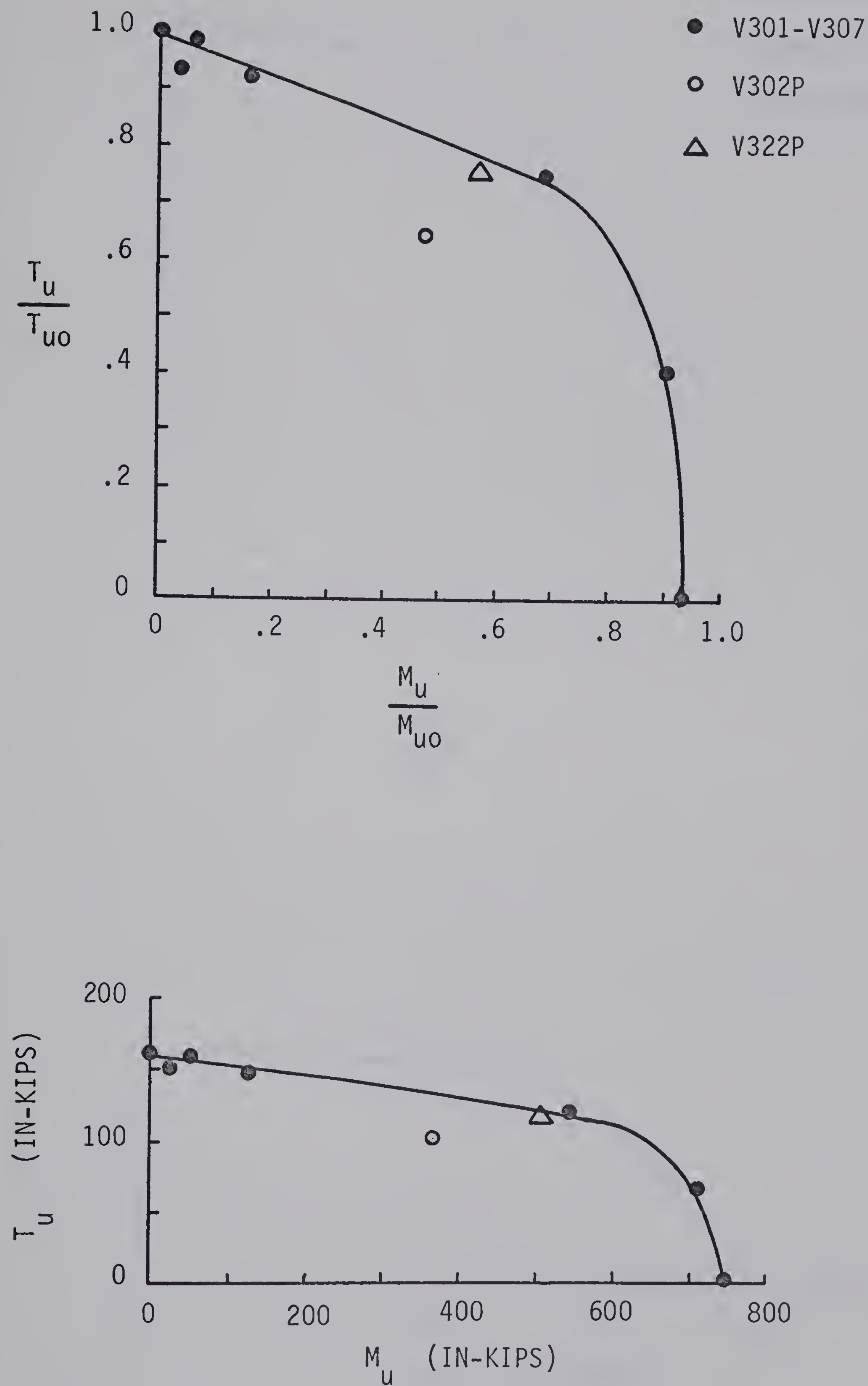
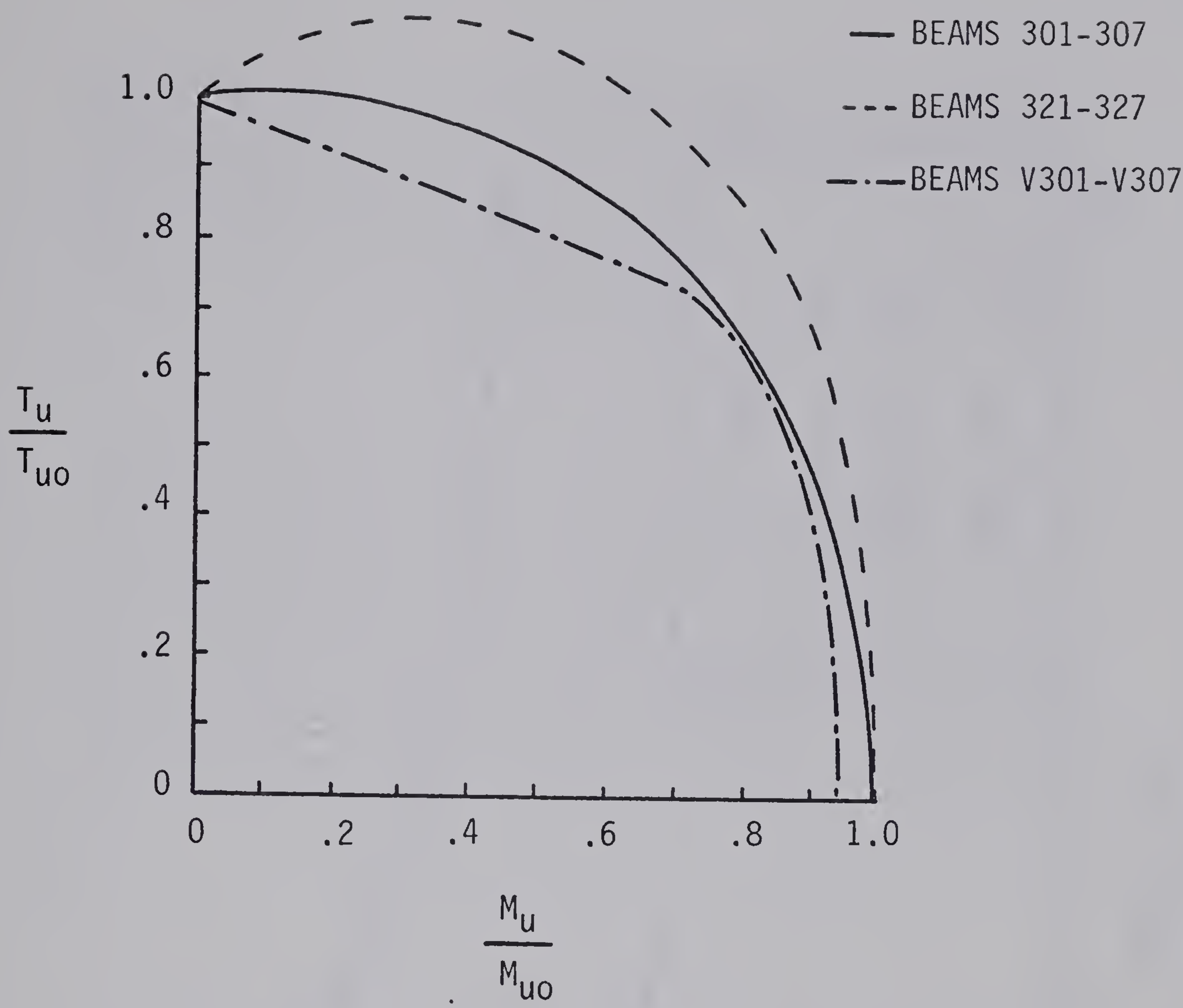


FIGURE 5.3B INTERACTION DIAGRAMS (GROUP III & IV BEAMS)



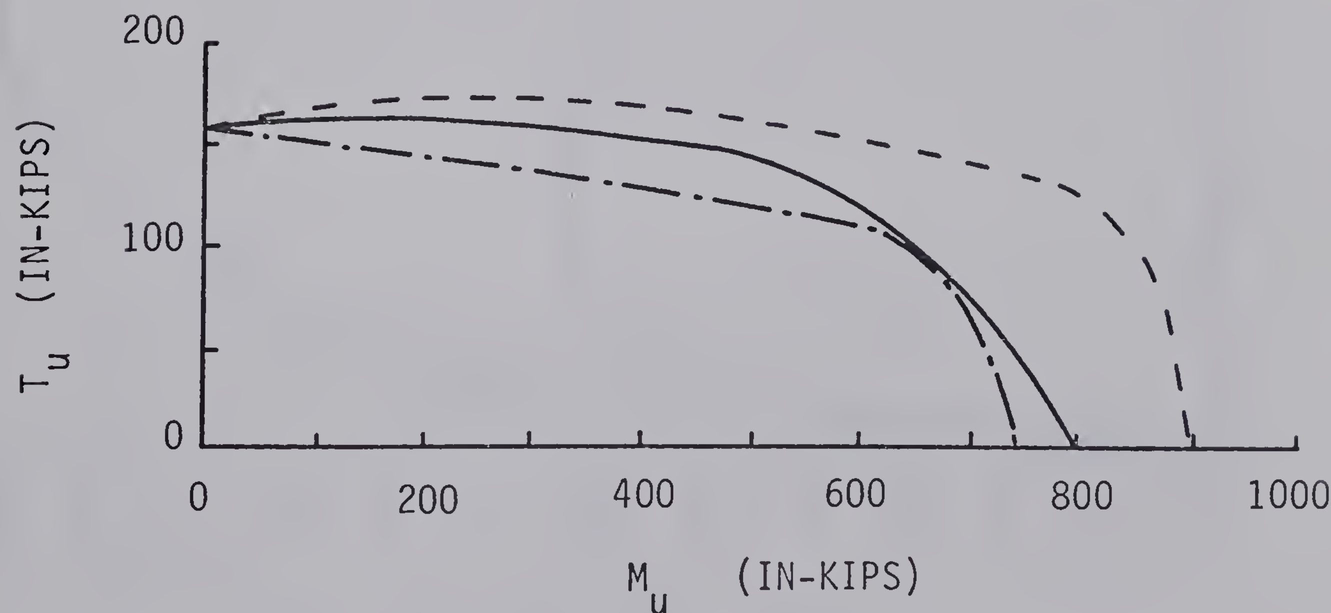


FIGURE 5.4 COMPARISON OF INTERACTION CURVES

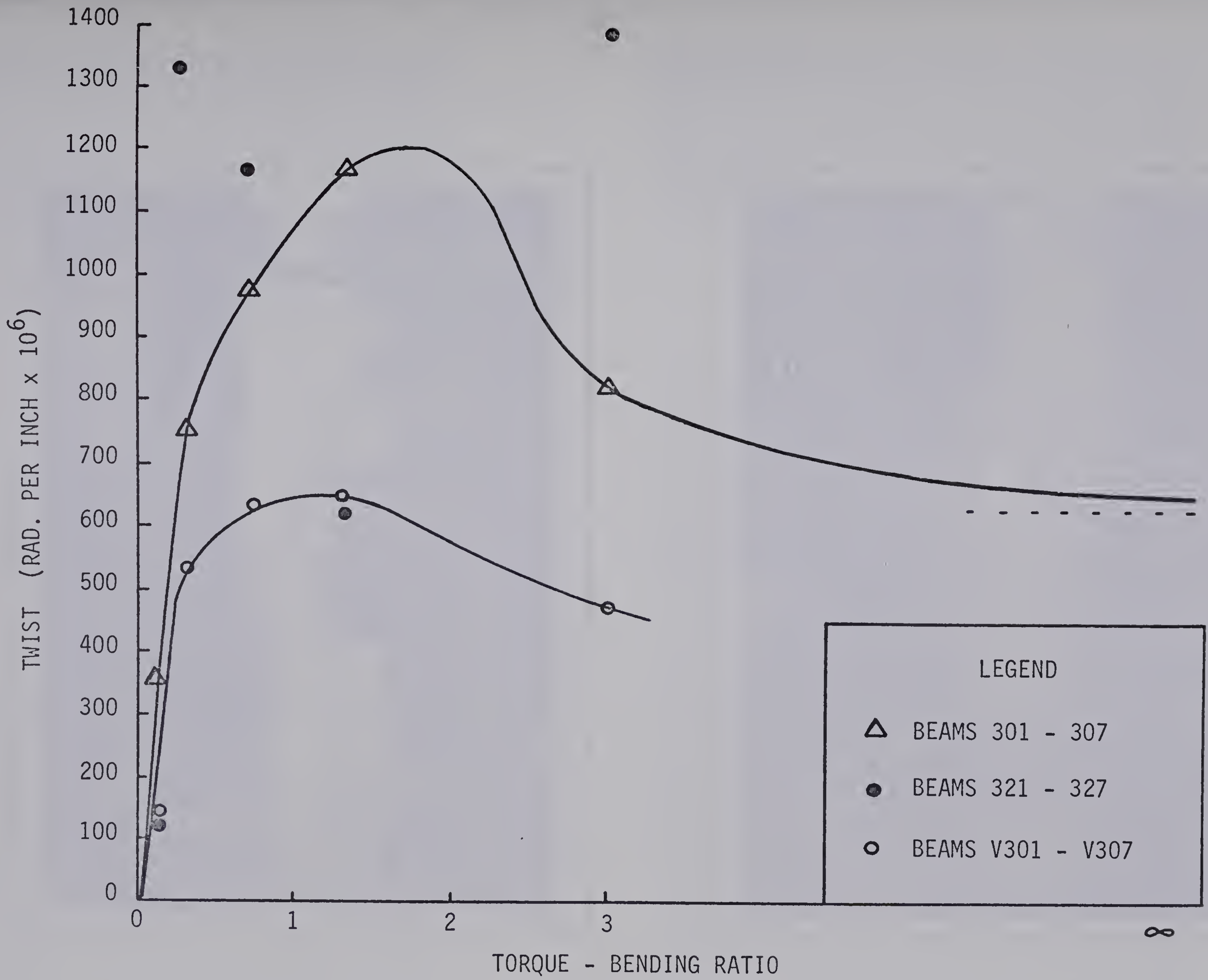
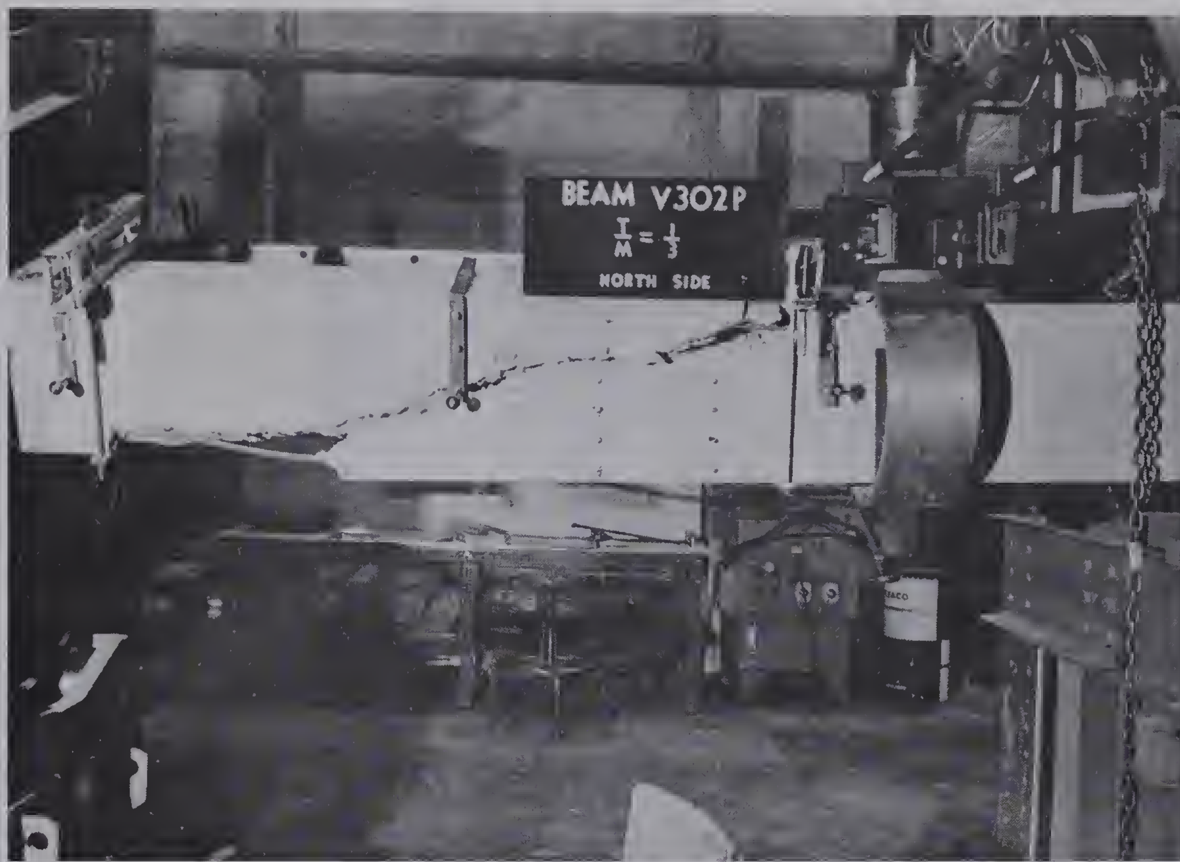


FIGURE 5.5 ULTIMATE TWIST VS. TORQUE - BENDING RATIO



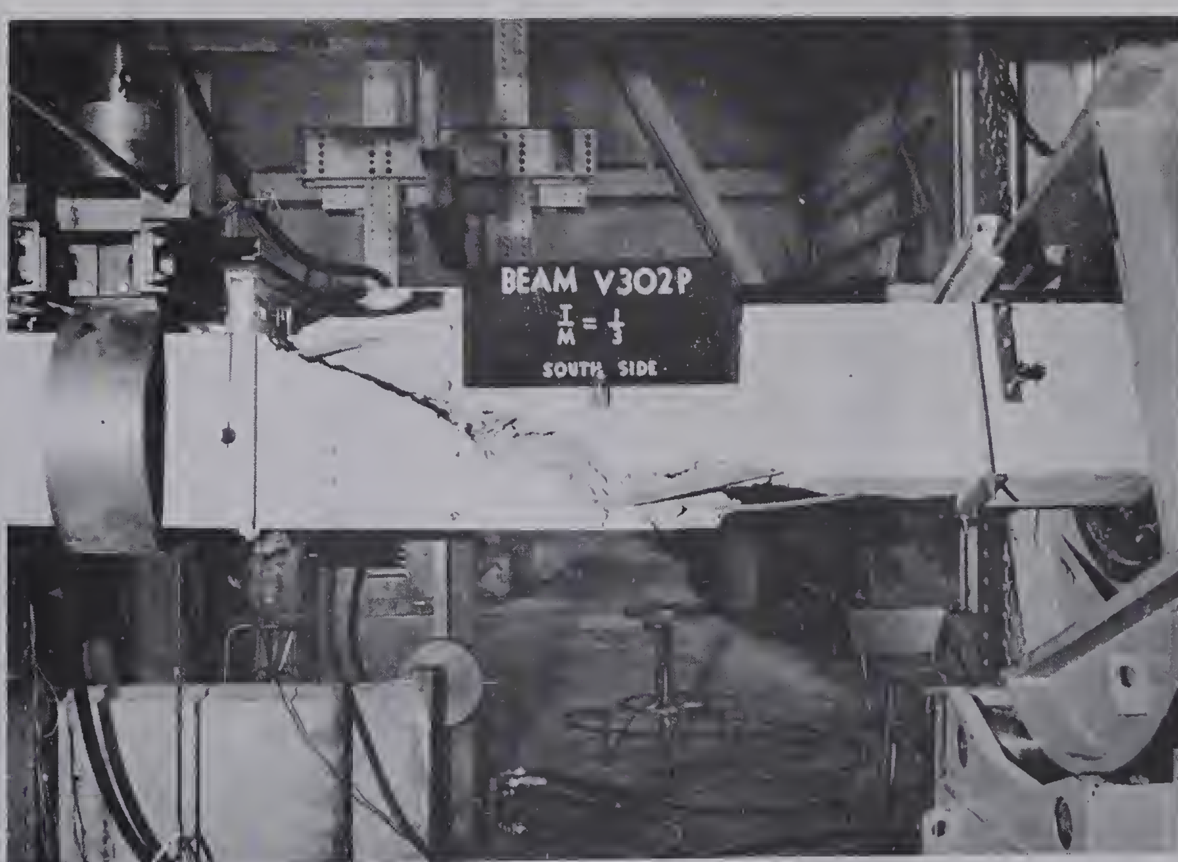


FIGURE 5.6 FAILURE SURFACE OF BEAM V302P

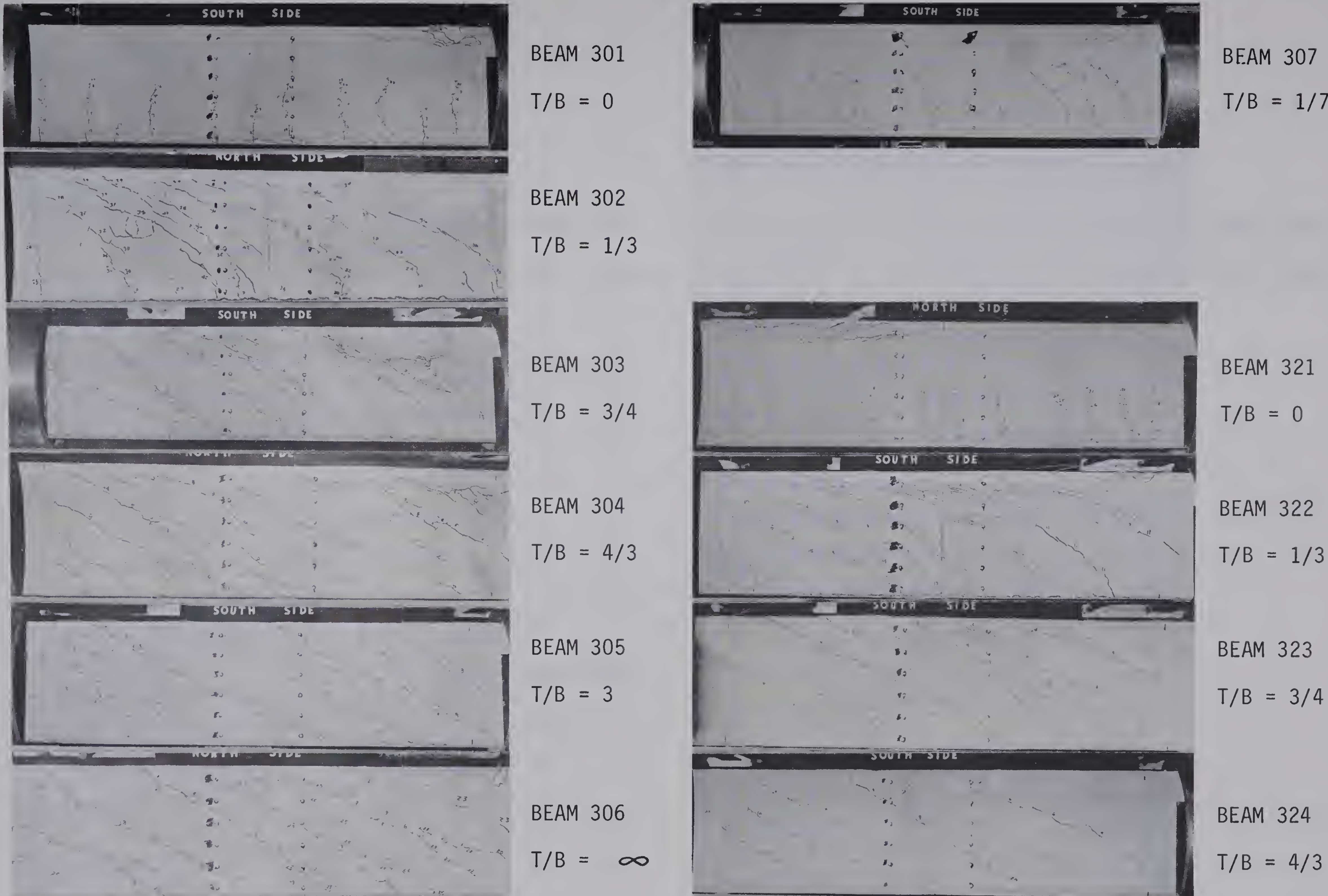
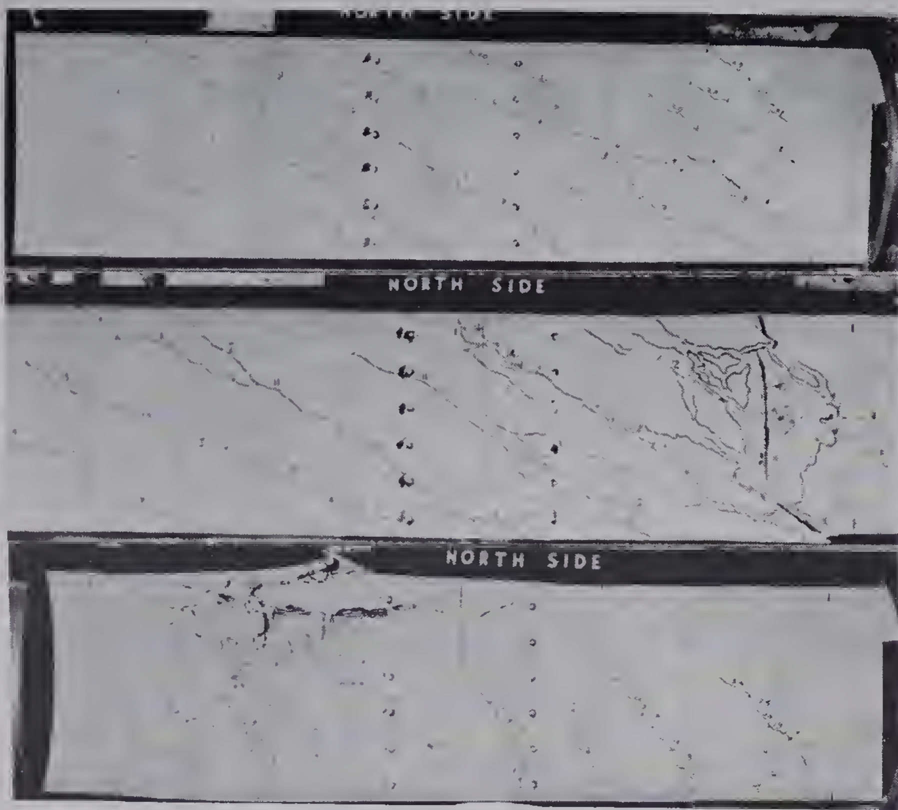


FIGURE 5.7 CRACK PATTERNS (BEAMS 301 to 307) AND (BEAMS 321 to 324)



BEAM 325

T/B = 3

BEAM 326

T/B = ∞

BEAM 327

T/B = 1/7

BEAM V303

T/B = 3/4

BEAM V304

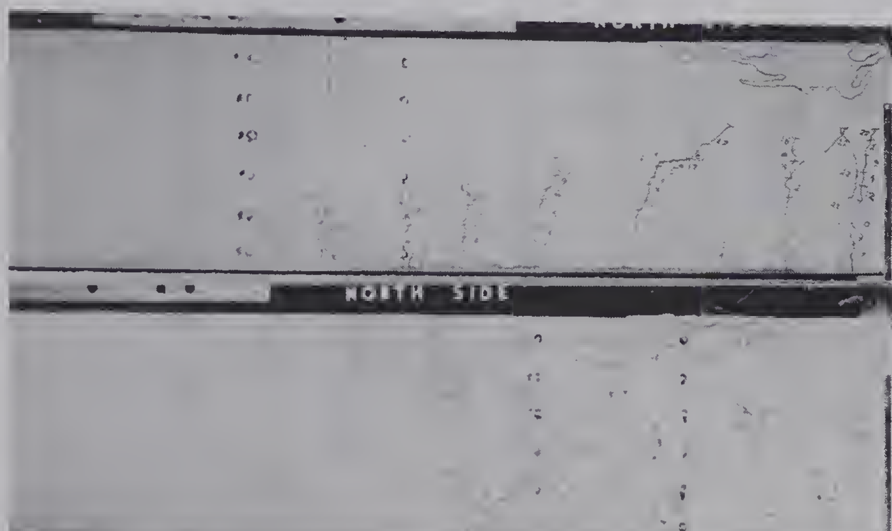
T/B = 4/3

BEAM V305

T/B = 3

BEAM V307

T/B = 1/7

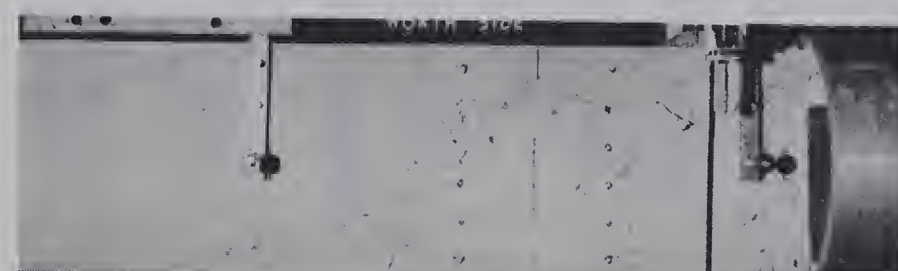


BEAM V301

T/B = 0

BEAM V302

T/B = 1/3



BEAM V322P

T/B = 1/3

FIGURE 5.8 CRACK PATTERNS (BEAMS 325 to 327) AND (BEAMS V301 to V307, V322P)

derived by Mukherjee and Warwaruk (3) :

$$\frac{T_u}{T_{uo}} - \left[2\frac{\sigma}{f'_c} + \frac{e}{d}\right]\frac{M_u}{M_{uo}} + \left[\frac{M_u}{M_{uo}}\right]^2 = 1.0$$

The interaction diagrams shown in Figures 5.1A, 5.1B, 5.2A, and 5.2B serve to indicate the beneficial effect which a small amount of bending moment has upon the ultimate torsional strength of a beam. This small increase in torsional strength of a beam is especially evident for beams which are eccentrically prestressed. Opposed to this, a small amount of applied torque had little or no effect upon the flexural capacity of the members. From these diagrams it is also evident that once the torsional strength had reached a peak, any further increase in the applied bending load caused a progressive deterioration in the torsional strength of the members.

5-5 BEHAVIOR UNDER COMBINED BENDING, TORSION, AND SHEAR

Typical crack patterns for the beams of Groups III and IV are shown in Figures 5.7 and 5.8. These beams exhibited a distinct similarity to those of Groups I and II in regard to the formation and progression of cracks in the members. However, whereas the cracks were evenly distributed along the gauge length in the case of Group I and II beams, the effect of the moment gradient present in the beams of Groups III and IV was to change the spacing of the cracks. Thus flexural cracks tended to be more concentrated in the region of higher bending moment whereas the torsional cracks generally were located in the region of lower bending moment, especially in the case of beams subjected to high torque to bending ratios.

Similar to the reinforcement of Group I and II beams, the transverse and longitudinal reinforcement of the Group III beams was instrumental in increasing the torsional strength of the members over and above the strength at cracking. This is illustrated in Table 4.2 which shows that only the transverse reinforcement of Beams V301 and V307 did not yield at ultimate. These two beams were subjected to torsion to bending moment ratios of 0 and 1:7 respectively.

Torque-twist curves for Group III and IV beams are illustrated in Figure A.3. As shown, the initial slope of all the curves are nearly the same, suggesting that the torsional stiffness prior to cracking is independent of torsion to bending ratios. In addition, the slopes are equal to those plotted for Groups I and II so that the initial torsional stiffness is also not affected significantly by flexural shear.

In the post-cracking stage the beams exhibited continued rotational capacity and a certain amount of ductility. However, a comparison to the torque-twist curves of Group I beams shown in Figure A.1 indicates that the rotational capacity as well as the ductility is less than that of the Group I beams. The presence of flexural shear in a member is therefore detrimental to both the torsional strength as well as the ductility of that member. Since the loss in ductility is proportionately larger than the loss in strength, the torsional stiffness in the post-cracking stage of a beam in which shear is present would necessarily be larger than that of a beam in which no shear is found.

The modes of failure for the Group III and IV beams were analogous to those of Group I and II beams. Depending upon the torque to

bending ratio the location of the failure plane varied throughout the gauge length for the members tested. As illustrated in Table 4.2, the distance from the plane of failure to the point of load application increased with increases in the torque to bending ratio. Such a failure surface is shown in Figure 5.6 which corresponds very closely to the warped failure surface described by Zia and GangaRao (4).

5-6 INTERACTION OF BENDING, TORSION, AND SHEAR

Combinations of three different types of loads such as bending, torsion, and shear necessarily require the use of a three-dimensional interaction surface in order to accurately describe their effect on one another. The use of two-dimensional diagrams describing such three-dimensional interactions is therefore only approximate, and is used in this investigation only because of ease of presentation.

The interaction diagrams for Groups III and IV are illustrated in Figure 5.3A and Figure 5.3B. As evidenced by Figure 5.3B, a small amount of bending moment has little or no effect on the ultimate torsional capacity of the specimens. Similarly, a small amount of torsional moment does not affect the bending capacity of a member significantly.

The overall effect of shear on a member is perhaps best illustrated by Figure 5.4 which shows a comparison between the interaction curves of Groups I, II, and III. As this figure indicates, the presence of shear in a member is clearly detrimental to the strength of that member. In every instance the ultimate capacity of beams with shear present was below that of correspondingly tested beams in which

no shear was present. In the case of combined bending and shear, a comparison between Table 4.1 and Table 4.2 shows that the effect of the shear was to reduce the ultimate capacity in flexure which the specimen was capable of sustaining.

Eliminating the transverse reinforcement in a beam is another factor which will reduce the ultimate strength of a beam. This is illustrated in Figures 5.3A and 5.3B which show the capacities of both V302P and V322P as being considerably lower than corresponding beams which did contain transverse reinforcement. Furthermore, at ultimate the amount of twist and deflection of these two specimens was also reduced. The eccentricity of prestress of Beam V322P tended to offset this loss in strength somewhat, but the capacity of this member was still below that of a corresponding concentrically prestressed member provided with transverse reinforcement.

A comparison of the experimental results to the interaction surface proposed by Mukherjee and Warwaruk (3) for beams subjected to bending, torsion, and shear is summarized in Table 5.2. The average interaction value I_u was 0.940 which indicates that the proposed interaction surface overestimated the capacities of the beams by six percent. This difference is not greatly significant and with additional test results the interaction surface relationships can be improved.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

6-1 INTRODUCTION

This chapter presents a summary of the results of tests performed on rectangular prestressed concrete beams subjected to bending, torsion, and shear. In addition, general conclusions and recommendations for further studies of this nature are included.

6-2 SUMMARY

For this study twenty-two rectangular prestressed concrete beams were tested. Fourteen beams were prestressed concentrically while the remaining beams were eccentrically prestressed at an eccentricity ratio of 0.167. An attempt was made to provide the same level of prestress for all the beams of the program.

Most beams exhibited two stages of behavior during the loading sequence. The first stage, designated the pre-cracking stage, was elastic and was terminated by major cracking in the beam. The cracking torque was usually indicated by an abrupt change in slope of either the torque-twist curve or the moment-deflection curve, or by a sudden jump in the strain readings for the transverse reinforcement. The second stage, or post-cracking stage, was characterized by a continued increase in the torsional strength of the beam, and considerable ductility due

to the transverse reinforcement was exhibited. A further increase in loading beyond the ultimate capacity of the member led to a rapid release of torsional load for the majority of the beams tested.

The test results have been presented in the form of tables, Torque-Twist curves, Moment-Deflection curves, and Interaction Diagrams. The interaction diagrams have been presented in dimensional and non-dimensional form for both cracking and ultimate load conditions. These diagrams summarize the general test results and serve to illustrate the effects of the torque to bending ratio, the types of prestressing, and the transverse shear.

6-3 CONCLUSIONS

The following conclusions are based on the test results of a limited number of rectangular prestressed concrete beams containing mild steel web reinforcement. These limitations should be considered when interpreting the conclusions.

From the test results it is concluded that:

(1) The torsional stiffness of a member prior to cracking is independent of flexural shear and the applied torsion to bending ratio. Also, the effect of the amount and spacing of the mild steel reinforcement as well as the prestressed reinforcement on the torsional stiffness is not significant.

(2) The ultimate strength and torsional stiffness which a beam exhibits in the post-cracking stage is decreased by an increase in the spacing of the transverse reinforcement.

(3) Eccentricity of prestress is beneficial to the ultimate capa-

city and ductility of a beam in comparison to a concentric type of prestress. This is true for a combined loading of torsion and bending, as well as for combined torsion, bending, and shear.

(4) An increase in the level of prestress increases the strength but reduces the twist and deflection of a beam at ultimate. Furthermore, raising the level of prestress increases the cracking torque of a member.

(5) A small amount of bending causes a small increase in the ultimate torsional strength of a beam subjected to a combined loading of bending and torsion. In the case of combined bending, torsion, and shear the beneficial effect of the presence of a small bending moment is eliminated by the transverse shear.

(6) The effect of flexural shear in a member is to reduce the strength as well as the ductility of that member.

(7) The transverse reinforcement yields at failure when the applied torsion to bending ratio is 3/4 or greater. This is true for combined bending and torsion as well as for combined bending, torsion, and shear.

(8) The non-prestressed transverse reinforcement serves to increase the rotational strength and ductility of a beam over and above that at cracking.

6-4 RECOMMENDATIONS

The following recommendations are made for the benefit of future investigations dealing with the interaction of bending, torsion, and shear in prestressed beams:

(1) The effect of the non-prestressed reinforcement on the strength and ductility of a beam should be more clearly isolated from the effect of the prestressing reinforcement.

(2) Beams with cross sections other than rectangular should be tested.

(3) The location of the failure plane in beams subjected to torsion, bending, and shear should be more accurately established.

(4) A prediction equation for the ultimate torsional strength of web reinforced, eccentrically prestressed beams should be derived.

(5) Recommendations for design of prestressed beams subjected to a combined loading of bending, torsion, and shear should be made.

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APPENDIX A

TEST RESULTS

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE			
				EAST	CENTER	WEST	1	2	3	4
0	-	12.2	-	0	0	0	0	0	0	0
1	-	40.5	-	10	15	10	- 2	8	- 8	8
2	-	81.0	-	25	30	25	- 1	3	- 12	4
3	-	121.5	-	45	50	40	0	4	- 15	4
4	-	162.0	-	60	70	60	- 2	4	- 15	3
5	-	243.0	-	90	105	90	- 2	3	- 19	3
6	-	364.5	-	145	175	150	- 10	0	- 17	- 2
7	-	445.5	-	190	225	195	- 15	- 10	- 19	- 11
8	-	486.0	-	225	265	230	- 24	- 16	- 39	- 3
9	-	594.0	-	350	435	365	+ 11	- 76	- 112	+ 226
10	-	654.8	-	450	550	465	169	- 94	- 134	437
11	-	681.8	-	500	615	515	283	- 96	- 139	452
12	-	702.0	-	545	670	555	374	- 97	- 139	493
13	-	715.5	-	575	700	585	423	- 95	- 143	514
14	-	729.0	-	615	745	615	306	- 102	- 152	545
15	-	735.8	-	630	770	630	253	- 105	- 155	556
16	-	742.5	-	655	790	645	241	- 107	- 145	575
17	-	749.3	-	670	805	665	227	- 104	- 149	585
18	-	756.0	-	690	830	675	218	- 102	- 150	601
19	-	762.8	-	710	850	700	200	- 100	- 156	612
20	-	769.5	-	725	880	715	189	- 104	- 149	634
21	-	776.3	-	750	900	735	162	- 101	- 159	642
22	-	783.0	-	775	925	755	153	- 103	- 154	655
23	-	789.8	-	795	955	780	137	- 102	- 153	663
24	-	796.5	-	825	990	800	129	- 100	-	-

TABLE A.1 BEAM 301

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	12.2	0	0	0	0	0	0	0	0
1	31.5	94.5	18	30	35	30	2	- 3	- 11	4
2	58.5	175.5	41	55	65	60	11	- 5	- 10	4
3	81.0	243.0	67	85	90	85	22	- 2	- 15	10
4	90.0	270.0	79	93	118	105	24	0	- 16	10
5	103.5	310.5	102	120	148	118	53	+ 12	- 17	21
6	117.0	351.0	130	145	175	145	95	54	- 17	25
7	126.0	378.0	168	175	210	170	101	248	0	38
8	128.3	384.8	184	180	215	178	98	290	+ 5	50
9	130.5	391.5	198	190	228	185	90	326	20	60
10	132.8	398.3	210	195	238	193	88	361	26	66
11	135.0	405.0	225	200	245	200	91	396	44	70
12	137.3	411.8	241	205	253	205	105	453	65	76
13	139.5	418.5	260	215	265	213	176	570	111	80
14	141.8	425.3	287	228	278	228	348	828	203	82
15	144.0	432.0	314	243	300	245	444	994	286	87
16	146.3	438.8	341	253	310	255	514	1094	388	88
17	148.5	445.5	367	263	325	265	606	1210	464	90
18	150.8	452.3	402	278	335	280	680	1349	650	91
19	153.0	459.0	456	295	363	288	746	1535	821	80
20	153.9	461.7	495	310	383	305	827	1633	934	110
21	154.8	464.4	540	330	398	325	911	1718	1048	195
22	156.6	469.8	627	355	440	358	1203	1837	1230	300
23	157.5	472.5	686	375	453	370	1342	1900	1317	349
24	158.4	475.2	746	393	475	383	1429	1877	1382	370
25	159.3	477.9	-	645	730	500	2200	1767	1122	348

TABLE A.2 BEAM 302

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	12.2	0	0	0	0	0	0	0	0
1	30.4	40.5	29	10	15	10	- 13	- 10	- 5	14
2	60.7	81.0	62	25	30	25	- 35	- 18	- 5	10
3	91.1	121.5	98	45	55	40	- 40	- 25	+ 5	22
4	101.2	135.0	118	50	60	45	- 39	- 22	17	47
5	111.4	148.5	135	50	60	50	- 30	- 16	30	120
6	121.5	162.0	157	60	65	50	+ 24	0	53	186
7	126.5	168.8	171	60	70	55	51	+ 10	61	229
8	131.6	175.5	186	65	75	60	81	22	76	274
9	136.7	182.3	203	70	75	70	124	32	91	316
10	141.7	189.0	224	75	85	75	194	40	111	398
11	146.8	195.8	275	85	95	85	349	34	124	695
12	148.8	198.5	313	90	100	95	415	31	175	795
13	150.8	201.2	383	95	115	95	495	142	292	850
14	151.8	202.5	464	100	125	105	642	429	386	912
15	152.9	203.9	548	110	135	115	817	660	469	939
16	153.4	204.5	629	120	140	120	934	798	517	1008
17	153.9	205.2	781	125	150	135	1025	945	580	1100
18	154.4	205.9	895	120	145	140	1134	1000	655	1159
19	154.9	206.3	970	125	155	145	1241	1140	774	1204

TABLE A.3 BEAM 303

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	12.2	0	0	0	0	0	0	0	0
1	36.0	27.0	29	5	5	0	1	1	13	2
2	72.0	54.0	71	10	10	5	14	4	29	- 1
3	108.0	81.0	125	15	15	15	43	57	68	+ 5
4	126.0	94.5	160	20	35	30	67	100	128	21
5	135.0	101.3	189	25	40	30	93	126	172	93
6	144.0	108.0	233	30	40	45	146	178	331	346
7	147.6	110.7	271	35	55	55	161	182	596	666
8	149.4	112.1	379	40	55	60	169	281	973	788
9	150.3	112.7	530	35	60	65	463	611	1448	887
10	151.2	113.4	589	35	65	65	860	770	1613	917
11	152.1	114.1	648	40	65	70	996	932	1767	947
12	153.0	114.8	713	40	60	75	1073	1150	2094	965
13	153.9	115.4	789	35	60	80	1156	1388	2797	986
14	154.8	116.1	886	40	65	80	1224	1555	3648	1022
15	155.7	116.8	1006	40	60	85	1341	1710	4454	1049
16	156.6	117.5	1167	40	65	95	1502	2027	5268	1085
17	156.4	118.1	1503	40	40	115	1428	2250	7218	946

TABLE A.4 BEAM 304

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE			
				EAST	CENTER	WEST	1	2	3	4
0	0	12.2	0	0	0	0	0	-	0	-
1	60.8	20.3	52	- 5	0	- 5	- 20	-	12	-
2	81.0	27.0	78	- 5	- 5	- 10	- 23	-	22	-
3	89.1	29.7	86	- 5	- 10	- 10	- 23	-	28	-
4	97.2	32.4	97	- 5	- 5	- 10	- 24	-	34	-
5	105.3	35.1	108	- 10	- 10	- 10	- 19	-	42	-
6	113.4	37.8	117	- 10	- 10	- 5	- 15	-	54	-
7	121.5	40.5	135	- 10	- 10	- 10	- 12	-	69	-
8	125.6	41.9	140	- 10	- 10	- 10	- 7	-	77	-
9	129.6	43.2	151	- 10	- 10	- 10	+ 3	-	93	-
10	133.7	44.6	160	- 5	- 5	- 10	10	-	109	-
11	137.7	45.9	170	- 5	- 5	- 5	21	-	129	-
12	141.8	47.3	179	0	0	0	31	-	148	-
13	145.8	48.6	194	0	- 5	- 5	40	-	174	-
14	149.9	50.0	200	0	0	- 5	55	-	190	-
15	153.9	51.3	214	- 5	0	0	73	-	226	-
16	155.9	52.0	238	0	0	0	90	-	291	-
17	158.0	52.7	284	0	+ 5	+ 5	95	-	754	-
18	160.0	53.3	635	+ 5	15	5	149	-	1730	-
19	162.0	54.0	705	10	10	15	278	-	1900	-
20	164.5	54.7	817	10	10	15	583	-	2090	-

TABLE A.5 BEAM 305

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 106	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	-	0				0	0	0	0
1	26.8	-	21				2	2	2	2
2	53.3	-	43				8	2	- 3	4
3	80.0	-	71				17	10	+ 4	13
4	106.5	-	103				50	30	17	30
5	117.0	-	117				60	45	25	42
6	127.7	-	133				78	67	29	50
7	133.0	-	143				85	90	34	60
8	135.7	-	152				92	94	45	64
9	138.4	-	157				96	113	21	71
10	141.0	-	163				106	125	48	74
11	143.7	-	168				110	136	56	80
12	146.4	-	173				119	158	63	82
13	149.2	-	179				124	175	78	88
14	154.7	-	189				166	198	98	95
15	160.2	-	200				168	247	136	118
16	162.9	-	211				188	268	159	136
17	165.6	-	219				201	293	204	184
18	168.3	-	232				220	320	250	207
19	170.9	-	254				233	446	411	274
20	172.3	-	278				258	572	528	310
21	173.6	-	316				286	665	789	410
22	174.9	-	381				412	866	996	510
23	175.5	-	514				544	1340	1278	657
24	177.9	-	632				686	1636	1622	1134
25	176.3	-	1195				1094	633	5492	1483

TABLE A.6 BEAM 306

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	12.2	0	0	0	0	0	0	0	0
1	9.7	67.5	6	20	25	15	- 3	12	0	4
2	19.3	135.0	16	45	55	45	- 5	11	0	0
3	28.9	202.5	24	70	90	70	- 10	14	- 2	- 6
4	38.6	270.0	35	105	125	100	- 16	10	- 3	- 8
5	44.4	310.5	44	120	145	115	- 18	8	- 5	- 12
6	50.2	351.0	54	145	170	140	- 22	8	- 11	- 15
7	55.9	391.5	63	175	205	170	- 22	3	- 19	- 18
8	59.8	418.5	71	200	210	190	- 28	10	- 33	- 14
9	63.7	445.5	86	230	270	220	- 20	18	- 48	- 14
10	67.5	472.5	103	260	310	245	+ 22	7	- 69	- 12
11	71.4	499.5	119	300	350	285	56	8	- 83	- 18
12	75.2	526.5	148	340	410	330	103	34	- 84	- 15
13	77.2	540.0	157	365	445	355	130	38	- 79	- 10
14	79.1	553.5	170	395	470	380	153	69	- 74	+ 4
15	81.0	567.0	183	420	505	405	170	82	- 69	13
16	82.9	580.5	202	455	535	440	210	102	- 52	25
17	84.9	594.0	216	480	575	465	244	123	- 38	45
18	86.8	607.5	240	520	615	500	286	151	- 17	62
19	88.7	621.0	260	555	665	530	326	180	- 4	81
20	90.7	634.5	286	600	710	580	368	212	+24	105
21	92.6	648.0	305	655	770	615	413	235	52	128
22	94.5	661.5	349	710	845	670	472	288	75	160

TABLE A.7 BEAM 307

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. $\times 10^6$	DEFLECTIONS IN. $\times 10^3$ EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	-	12.2	-	0	0	0			0	
1	-	67.5	-	15	15	20			0	
2	-	135.0	-	40	45	45			2	
3	-	202.5	-	65	75	70			2	
4	-	270.0	-	95	110	100			2	
5	-	337.5	-	115	135	120			2	
6	-	405.0	-	145	175	155			2	
7	-	486.0	-	195	230	200			8	
8	-	499.5	-	200	235	205			8	
9	-	567.0	-	245	295	250			8	
10	-	621.0	-	300	355	305			4	
11	-	675.0	-	360	435	365			- 18	
12	-	702.0	-	395	475	405			- 35	
13	-	729.0	-	435	525	445			- 45	
14	-	756.0	-	480	580	490			- 58	
15	-	796.5	-	545	665	570			- 76	
16	-	810.0	-	570	690	590			- 83	
17	-	823.5	-	595	725	620			- 87	
18	-	837.0	-	625	755	645			- 93	
19	-	850.5	-	650	790	675			- 97	
20	-	864.0	-	690	825	705			- 103	
21	-	877.5	-	720	875	750			- 106	
22	-	891.0	-	775	925	800			- 109	
23	-	904.5	-	820	980	820			- 113	

TABLE A.8 BEAM 321

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	12.2	0	0	0	0	0		0	0
1	13.5	40.5	10	15	10	10	1		2	5
2	27.0	81.0	21	30	30	25	1		2	1
3	40.5	121.5	35	45	45	40	1		2	2
4	54.0	162.0	51	65	65	60	2		3	5
5	67.5	202.5	65	85	85	75	5		5	5
6	81.0	243.0	84	100	110	95	7		5	4
7	90.0	270.0	97	115	130	110	10		4	10
8	99.0	297.0	113	125	145	120	16		24	10
9	108.0	324.0	133	145	160	135	30		38	15
10	117.0	351.0	157	155	180	150	61		57	26
11	126.0	378.0	189	175	200	170	105		86	35
12	135.0	405.0	237	200	235	190	146		190	55
13	139.5	418.5	303	215	250	215	187		414	74
14	144.0	432.0	402	240	280	235	515		768	100
15	148.5	445.5	578	270	315	255	905		853	161
16	150.8	452.3	713	280	335	280	1009		908	181
17	153.0	459.0	838	305	360	305	1100		983	193
18	154.2	465.8	1330	445	455	455	1095		1094	197

TABLE A.9 BEAM 322

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. $\times 10^6$	DEFLECTIONS IN. $\times 10^3$ EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	12.2	0	0	0	0	0	0	0	0
1	20.3	27.0	13	10	5	5	2	1	6	3
2	40.5	54.0	32	20	15	15	5	2	12	5
3	60.7	81.0	51	30	20	25	9	2	8	3
4	81.0	108.0	73	35	35	30	15	5	18	7
5	91.1	121.5	86	40	45	40	20	12	20	10
6	101.2	135.0	102	45	50	45	27	17	23	15
7	106.3	141.8	108	50	55	50	31	21	37	20
8	111.4	148.5	114	50	60	55	36	28	47	20
9	116.4	155.3	122	50	60	55	40	38	43	28
10	121.5	162.0	133	60	65	55	46	41	67	27
11	126.5	168.8	140	65	70	60	54	54	80	43
12	131.6	175.5	149	60	70	60	66	68	99	48
13	136.7	182.3	160	65	75	65	75	83	121	59
14	141.7	189.0	156	65	80	65	87	105	140	72
15	146.8	195.8	184	70	85	75	106	129	175	84
16	151.9	202.5	197	75	90	75	130	156	227	101
17	155.9	207.9	219	80	90	80	170	186	292	120
18	160.0	213.3	243	85	95	80	334	220	533	172
19	164.0	218.7	595	90	110	95	941	1072	1855	586
20	165.0	220.0	790	95	115	110	1156	1461	3110	950
21	166.0	221.4	867	95	115	110	1130	1592	3360	1072
22	167.0	222.8	917	95	115	115	1110	1683	3568	1139
23	168.1	224.1	958	100	115	120	1121	1767	3762	1185
24	170.1	226.8	1029	100	115	125	1151	1894	4013	1253
25	172.1	229.5	1160	105	125	130	1214	2320	4356	1309
26	147.5	232.2	1886	125	95	235	1226	3318	4546	1247

TABLE A.10 BEAM 323

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	12.2	0	0	0	0	0	0	0	0
1	36.0	27.0	24	5	0	0	- 3	1	- 3	- 1
2	54.0	40.5	52	5	10	5	- 4	2	- 5	- 4
3	72.0	54.0	73	15	10	10	- 4	7	- 3	- 5
4	90.0	67.5	95	20	20	15	- 2	19	0	- 2
5	108.0	81.0	122	20	25	20	+ 5	38	+ 10	+ 7
6	117.0	87.8	140	25	30	25	9	52	18	12
7	126.0	94.5	154	25	30	25	15	67	33	20
8	135.0	101.3	178	20	30	30	26	88	53	50
9	140.4	105.3	194	30	35	30	34	102	65	44
10	144.0	108.0	203	30	35	30	39	116	88	50
11	145.8	109.4	213	30	35	30	45	127	101	50
12	147.6	110.7	218	30	40	30	50	136	114	56
13	149.4	112.1	219	30	40	30	54	146	130	63
14	151.2	113.4	230	30	35	30	60	155	151	66
15	153.0	114.8	237	30	40	35	61	169	186	72
16	154.8	116.1	243	30	40	30	66	186	222	77
17	156.6	117.5	244	25	40	30	80	202	304	86
18	158.4	118.8	256	30	40	30	85	243	450	102
19	160.2	120.2	283	30	40	35	96	340	680	115
20	162.0	121.5	624	25	25	35	296	1268	1420	641

TABLE A.11 BEAM 324

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	12.2	0	0	0	0	0	0	0	0
1	60.8	20.3	51	0	0	- 5	5	3	19	7
2	81.0	27.0	63	0	- 5	- 10	11	7	25	14
3	89.1	29.7	89	- 5	0	- 10	14	12	30	20
4	97.2	32.4	102	- 5	- 5	- 10	17	15	33	20
5	105.3	35.1	113	- 10	- 5	- 10	23	27	43	26
6	113.4	37.8	124	- 10	- 10	- 15	31	44	50	38
7	121.5	40.5	141	- 15	- 5	- 15	42	55	59	39
8	125.6	41.9	149	- 10	- 10	- 20	48	69	68	45
9	129.6	43.2	156	- 10	- 5	- 20	55	84	75	58
10	133.7	44.6	165	- 15	- 10	- 20	67	107	85	65
11	137.7	45.9	173	- 15	- 10	- 20	88	130	97	71
12	141.8	47.3	184	- 15	- 10	- 20	103	155	113	76
13	145.8	48.6	192	- 15	- 10	- 20	118	192	128	90
14	149.9	50.0	203	- 15	- 15	- 20	138	240	143	105
15	153.9	51.3	213	- 15	- 15	- 25	160	310	171	107
16	155.9	52.0	225	- 15	- 10	- 20	190	355	214	128
17	158.0	52.7	237	- 20	- 15	- 25	227	431	302	145
18	160.0	53.3	267	- 20	- 20	- 25	305	743	416	355
19	162.0	54.0	527	- 25	- 40	- 30	474	1960	1017	738
20	164.0	54.7	851	- 35	- 70	- 40	1580	3701	1309	892
21	166.1	55.4	1008	- 40	- 75	- 45	1760	2950	1411	948
22	168.1	56.0	1124	- 50	- 90	- 45	1877	2590	1550	996
23	170.1	56.7	1225	- 45	- 105	- 40	1988	2470	1820	1023
24	172.1	57.4	1395	- 45	- 110	- 40	2217	2235	2610	1071

TABLE A.12 BEAM 325

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	-	0				0	0	0	
1	26.8	-	25				2	7	3	
2	53.3	-	53				5	12	12	
3	80.0	-	81				15	10	26	
4	106.5	-	119				35	52	52	
5	117.0	-	140				55	65	70	
6	127.7	-	162				82	90	104	
7	133.0	-	176				102	107	118	
8	138.3	-	187				113	128	143	
9	143.7	-	222				131	164	172	
10	149.2	-	254				146	307	280	
11	152.5	-	1143				1236	1784	4610	
12	153.1	-	1243				1250	2020	5910	
13	153.6	-	1359				1306	2480	7052	
14	154.2	-	1479				1308	2856	7977	
15	143.1	-	1956				1173	3150	-	

TABLE A.13 BEAM 326

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	12.2	0	0	0	0	0	0	0	0
1	28.9	202.5	3	75	85	70	- 12	- 17	- 5	2
2	50.2	351.0	21	130	165	135	- 27	- 24	- 4	- 3
3	67.5	472.5	46	200	245	200	- 46	- 42	- 6	- 5
4	81.0	567.0	89	275	340	285	- 50	+ 11	+ 18	- 2
5	87.7	621.0	125	340	410	345	+ 30	73	106	+ 41
6	90.7	634.5	138	355	435	360	81	83	148	55
7	92.6	648.0	152	385	460	380	115	88	198	65
8	94.5	661.5	175	400	490	405	151	90	250	84
9	96.4	675.0	224	440	540	445	274	117	424	143
10	97.4	681.8	248	455	560	470	330	124	496	163
11	98.4	688.5	271	475	580	490	388	135	578	183
12	99.3	695.3	279	485	595	510	412	150	601	186
13	100.3	702.0	290	500	620	525	426	157	617	183
14	102.2	715.5	298	520	645	545	444	168	638	191
15	104.2	729.0	316	540	665	570	460	178	664	202
16	106.1	742.5	325	570	700	600	471	191	689	212
17	108.0	756.0	341	595	735	635	487	205	704	215
18	110.0	769.5	346	625	765	665	495	212	714	213
19	111.9	783.0	359	655	815	705	500	217	728	213
20	113.8	796.5	371	695	865	745	506	234	741	216
21	115.7	810.0	384	740	910	810	507	244	752	219
22	117.7	823.5	385	805	1005	905	482	250	750	234
23	119.6	837.0	395	875	1085	1015	487	255	761	241
24	121.5	850.5	408	930	1170	1085	489	264	770	254
25	123.5	864.0	429	1015	1260	1180	483	284	766	257

TABLE A.14 BEAM 327

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	-	6.5	0.09	-	0	0	0		0	0	0
1	-	72.0	1.00	-	10	15	20		0	- 3	0
2	-	144.0	2.00	-	20	35	40		0	- 5	0
3	-	216.0	3.00	-	30	55	65		- 3	- 7	0
4	-	264.0	3.67	-	30	65	80		- 3	- 7	- 3
5	-	312.0	4.33	-	40	85	100		- 3	- 9	- 3
6	-	360.0	5.00	-	50	95	115		- 4	- 10	- 6
7	-	408.0	5.67	-	60	115	135		- 5	- 12	- 7
8	-	456.0	6.33	-	65	135	160		- 5	- 16	- 8
9	-	504.0	7.00	-	75	155	195		- 6	- 19	- 20
10	-	552.0	7.67	-	85	190	235		- 9	- 24	- 24
11	-	576.0	8.00	-	95	210	260		- 10	- 30	- 30
12	-	600.0	8.33	-	105	230	285		- 11	- 30	- 37
13	-	624.0	8.67	-	115	250	310		- 16	- 34	- 51
14	-	648.0	9.00	-	115	270	335		- 20	- 31	- 65
15	-	672.0	9.33	-	125	290	370		- 17	- 33	- 80
16	-	696.0	9.67	-	135	320	400		- 17	- 33	- 90
17	-	720.0	10.00	-	145	345	435		- 19	- 33	- 102
18	-	744.0	10.33	-	155	370	470		- 20	- 35	- 112
19	-	768.0	10.67	-	170	400	515		- 21	- 35	- 117
20	-	792.0	11.00	-	185	440	565		- 23	- 35	- 122
21	-	816.0	11.33	-	200	475	625		- 26	- 30	- 126
22	-	840.0	11.67	-	215	530	690		- 32	- 8	- 141

TABLE A.15 BEAM V301

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	6.5	0.09	0	0	0	0			0	0
1	13.0	72.0	1.00	13	10	15	20			- 3	- 1
2	26.0	144.0	2.00	33	15	30	40			- 4	- 5
3	39.0	216.0	3.00	47	25	50	60			- 7	- 7
4	52.0	288.0	4.00	61	35	70	85			- 6	- 14
5	65.0	360.0	5.00	81	45	95	115			- 7	- 22
6	73.7	408.0	5.67	97	55	115	145			- 5	- 22
7	82.4	456.0	6.33	113	55	145	170			- 5	- 33
8	91.0	504.0	7.00	138	75	165	210			+ 4	- 1
9	95.4	528.0	7.33	156	80	185	230			15	+ 89
10	99.7	552.0	7.67	172	90	205	260			34	150
11	104.0	576.0	8.00	197	100	230	290			49	216
12	108.4	600.0	8.33	233	110	255	320			238	330
13	110.5	612.0	8.50	263	115	275	350			375	397
14	112.7	624.0	8.67	296	125	290	370			569	450
15	114.9	636.0	8.84	340	130	310	400			760	540
16	117.0	648.0	9.00	396	135	330	425			984	633
17	119.2	660.0	9.17	461	145	360	460			1173	722
18	119.6	662.4	9.20	536	160	405	510			1450	828

TABLE A.16 BEAM V302

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	DEFLECTIONS IN. x 10^3 CENTER	DEFLECTIONS IN. x 10^3 WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	GAUGE 2	GAUGE 3	GAUGE 4
0	0	6.5	0.09	0	0	0	0	0	0		0
1	19.5	48.0	0.67	15	5	5	5	2	6		0
2	39.0	96.0	1.33	40	5	15	20	9	8		- 2
3	58.5	144.0	2.00	61	10	25	35	13	21		- 5
4	78.0	192.0	2.67	82	15	40	50	38	37		- 2
5	87.7	216.0	3.00	94	15	50	60	45	57		0
6	97.5	240.0	3.33	111	15	55	65	77	72		+ 10
7	107.2	264.0	3.67	131	20	60	80	99	84		10
8	117.0	288.0	4.00	153	25	75	90	166	118		18
9	126.7	312.0	4.33	185	30	80	105	414	200		- 15
10	131.6	324.0	4.50	219	30	90	115	635	295		- 24
11	136.5	336.0	4.67	296	25	105	130	1074	743		- 35
12	141.4	348.0	4.83	390	35	115	140	1282	1193		+ 53
13	146.2	360.0	5.00	633	45	145	165	1900	1729		162

TABLE A.17 BEAM V303

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	6.5	0.09	0	0	0	0	0		0	0
1	17.3	24.0	0.33	13	0	5	5	2		0	4
2	34.7	48.0	0.67	32	0	10	10	6		- 2	2
3	52.0	72.0	1.00	49	5	20	15	9		- 1	0
4	69.3	96.0	1.33	71	10	10	15	15		+ 9	5
5	78.0	108.0	1.50	79	10	15	20	20		15	7
6	86.7	120.0	1.67	90	5	25	15	25		24	12
7	95.3	132.0	1.83	104	10	30	25	39		33	17
8	104.0	144.0	2.00	117	15	30	25	59		46	28
9	112.7	156.0	2.17	133	10	35	35	111		61	54
10	121.3	168.0	2.33	153	10	35	40	275		76	88
11	130.0	180.0	2.50	175	10	45	40	553		132	130
12	138.6	192.0	2.67	233	15	50	45	1367		228	169
13	142.1	196.8	2.73	267	10	55	50	1750		303	197
14	145.6	201.6	2.80	313	10	60	55	2375		405	240
15	149.0	206.4	2.87	406	5	65	60	3900		497	296
16	152.5	211.2	2.93	490	10	65	65	5670		933	394
17	156.0	216.0	3.00	640	5	80	80	7203		1194	954

TABLE A.18 BEAM V304

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	6.5	0.09	0	0	0	0	0	0	0	-
1	19.5	12.0	0.17	21	0	0	0	- 3	- 5	10	-
2	39.0	24.0	0.33	39	0	0	5	- 8	- 15	15	-
3	46.8	28.8	0.40	46	5	0	0	- 14	- 22	15	-
4	54.6	33.6	0.47	53	5	5	5	- 17	- 23	18	-
5	62.4	38.4	0.53	61	0	5	0	- 18	- 26	21	-
6	70.2	43.2	0.60	69	0	0	5	- 4	- 22	25	-
7	78.0	48.0	0.67	81	0	5	5	+ 4	- 20	32	-
8	85.8	52.8	0.73	90	- 5	5	5	18	- 19	40	-
9	93.6	57.6	0.80	100	0	5	5	33	- 8	50	-
10	101.4	62.4	0.87	113	0	5	5	51	+ 1	58	-
11	109.2	67.2	0.93	125	- 5	5	5	71	13	70	-
12	117.0	72.0	1.00	139	0	5	5	95	31	97	-
13	124.8	76.8	1.07	164	0	5	10	908	53	121	-
14	128.7	79.2	1.10	194	- 5	15	10	1674	89	145	-
15	132.6	81.6	1.13	219	- 5	10	10	2091	94	162	-
16	136.5	84.0	1.17	242	- 5	10	10	2490	111	188	-
17	140.4	86.4	1.20	265	- 10	10	10	3191	148	218	-
18	144.3	88.8	1.23	339	- 15	15	15	4352	1008	274	-
19	148.2	91.2	1.27	469	- 20	15	15	5018	1485	277	-

TABLE A.19 BEAM V305

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	6.5	0.09	0	0	0	0	0	0	0	0
1	5.6	72.0	1.00	7	10	15	20	- 2	- 6	- 2	3
2	11.2	144.0	2.00	11	15	35	40	+ 1	- 2	+ 4	5
3	16.7	216.0	3.00	14	25	55	60	1	- 6	1	3
4	22.3	288.0	4.00	19	40	75	85	3	- 5	3	5
5	26.0	336.0	4.67	24	45	90	105	3	- 5	- 1	1
6	29.7	384.0	5.33	26	50	105	120	4	- 8	- 2	3
7	33.4	432.0	6.00	32	60	125	150	5	- 5	- 4	2
8	37.2	480.0	6.67	36	70	145	175	9	- 7	- 5	2
9	40.9	528.0	7.33	43	85	175	215	10	- 7	- 10	4
10	44.6	576.0	8.00	50	95	205	255	13	- 7	- 20	- 10
11	48.3	624.0	8.67	58	110	245	305	15	- 12	- 29	- 12
12	50.2	648.0	9.00	64	120	265	330	16	- 19	- 32	- 10
13	52.0	672.0	9.33	69	130	290	365	15	- 35	- 35	- 1
14	53.9	696.0	9.67	78	135	315	395	17	- 21	- 38	+ 8
15	55.7	720.0	10.00	83	145	340	430	20	- 18	- 42	21
16	57.6	744.0	10.33	90	155	360	460	23	- 18	- 46	41
17	59.4	768.0	10.67	97	170	395	500	23	- 24	- 50	64
18	61.3	792.0	11.00	108	180	425	540	28	- 19	- 60	117
19	63.2	816.0	11.33	117	195	460	595	29	- 20	- 56	175
20	65.0	840.0	11.67	135	210	510	655	33	- 28	- 58	271

TABLE A.20 BEAM V307

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	6.5	0.09	0	0	0	0				
1	8.7	48.0	0.67	3	10	10	15				
2	17.3	96.0	1.33	10	15	20	35				
3	26.0	144.0	2.00	18	20	35	45				
4	34.7	192.0	2.67	25	25	50	60				
5	43.3	240.0	3.33	33	30	65	70				
6	47.7	264.0	3.67	43	35	70	75				
7	52.0	288.0	4.00	47	40	80	85				
8	56.3	312.0	4.33	53	45	85	95				
9	60.7	336.0	4.67	57	45	95	110				
10	62.8	348.0	4.83	61	50	100	115				
11	65.0	360.0	5.00	64	50	105	125				
12	67.2	372.0	5.17	67	50	105	125				
13	69.4	384.0	5.33	70	50	115	130				
14	71.5	396.0	5.50	76	50	115	135				
15	73.7	408.0	5.67	78	50	120	140				
16	75.9	420.0	5.83	83	55	125	150				
17	78.0	432.0	6.00	85	60	130	160				
18	80.2	444.0	6.17	90	60	135	160				
19	82.4	456.0	6.33	93	65	145	170				
20	86.7	480.0	6.67	106	75	160	190				
22	91.0	504.0	7.00	121	80	185	220				
23	93.2	516.0	7.17	135	85	195	240				
24	95.4	528.0	7.33	149	90	205	255				
25	99.7	552.0	7.67	185	100	235	295				
26	101.9	564.0	7.83	208	105	250	315				
27	104.0	576.0	8.00	-	-	-	-				

TABLE A.21 BEAM V302P

LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RADIANS PER IN. x 10^6	DEFLECTIONS IN. x 10^3 EAST	CENTER	WEST	REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE 1	2	3	4
0	0	6.5	0.09	0	0	0	0				
1	13.0	72.0	1.00	10	5	10	20				
2	26.0	144.0	2.00	22	15	30	40				
3	39.0	216.0	3.00	35	25	50	65				
4	52.0	288.0	4.00	47	35	70	85				
5	65.0	360.0	5.00	61	40	90	115				
6	73.7	408.0	5.67	71	50	105	125				
7	82.4	456.0	6.33	85	60	120	150				
8	86.7	480.0	6.67	90	65	130	165				
9	91.0	504.0	7.00	99	60	140	175				
10	95.4	528.0	7.33	104	65	155	190				
11	99.7	552.0	7.67	108	75	165	205				
12	104.0	576.0	8.00	125	80	175	220				
13	106.2	588.0	8.17	129	85	190	235				
14	108.4	600.0	8.33	139	85	195	245				
15	110.5	612.0	8.50	154	95	210	265				
16	112.7	624.0	8.67	164	95	220	275				
17	114.9	636.0	8.83	172	100	230	290				
18	117.0	648.0	9.00	261	105	255	315				

TABLE A.22 BEAM V322P

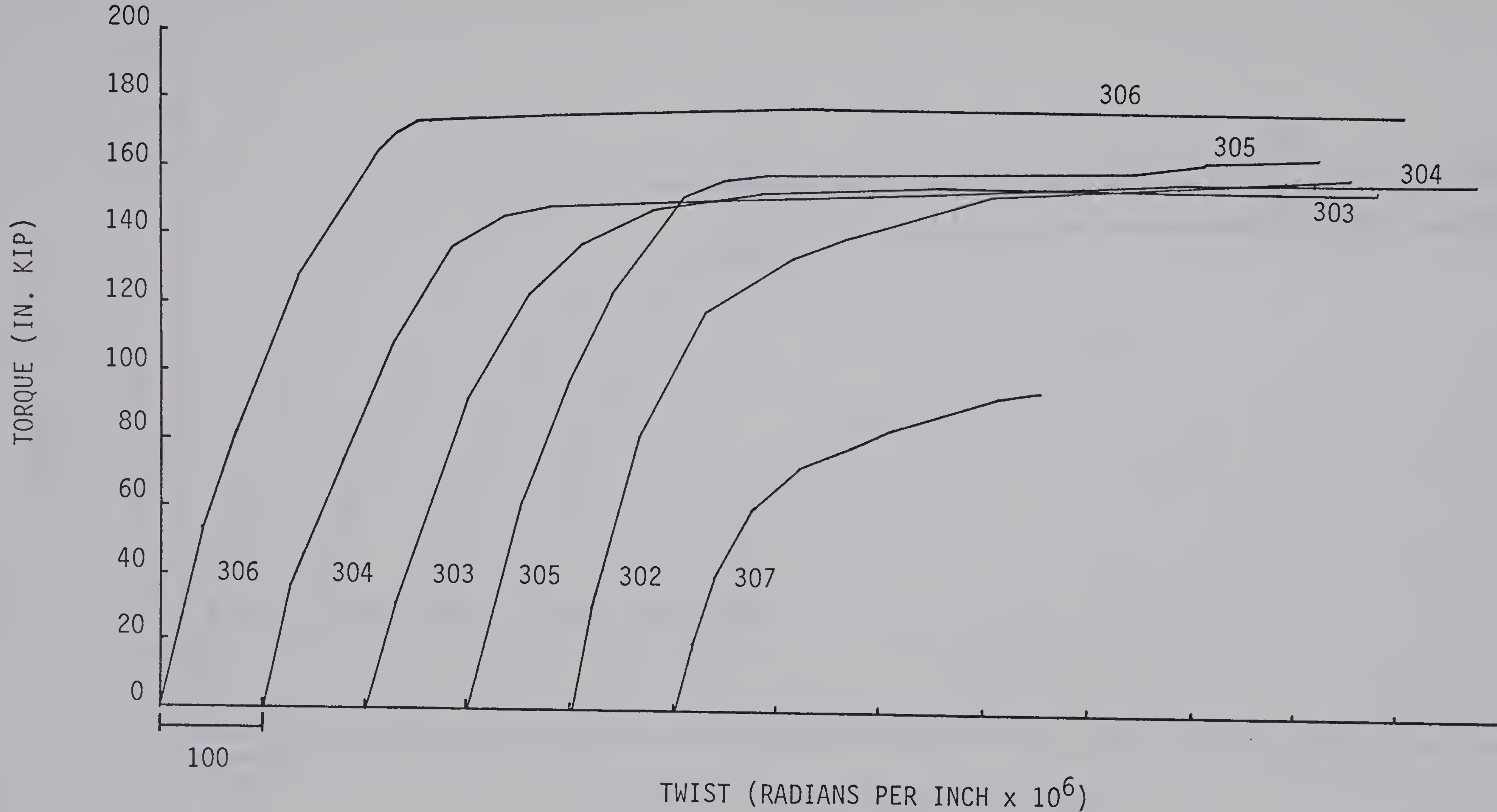

FIG. A.1 TORQUE-TWIST CURVES (BEAMS 302 to 307)

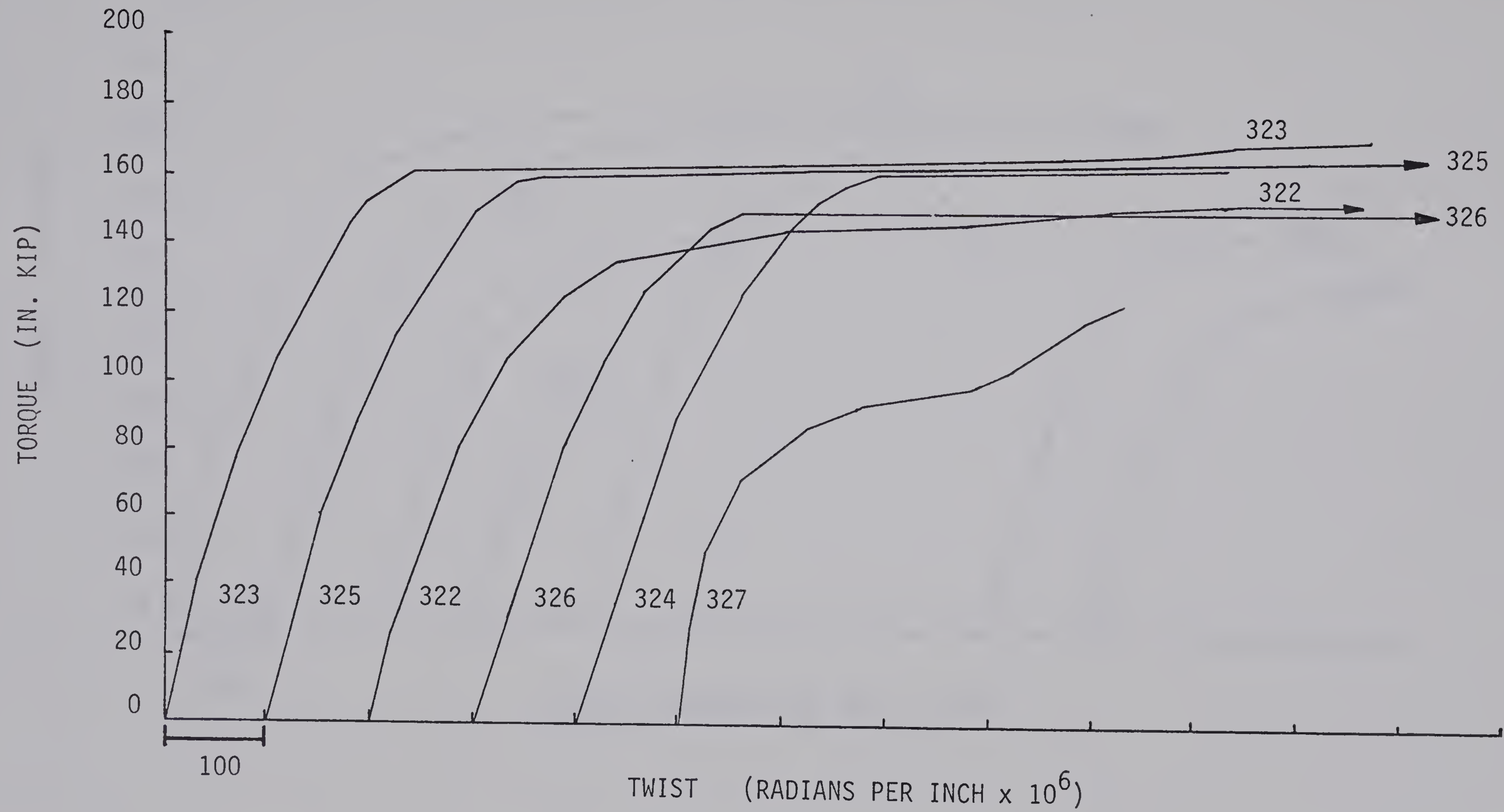

FIG. A.2 TORQUE - TWIST CURVES (BEAMS 322 to 327)

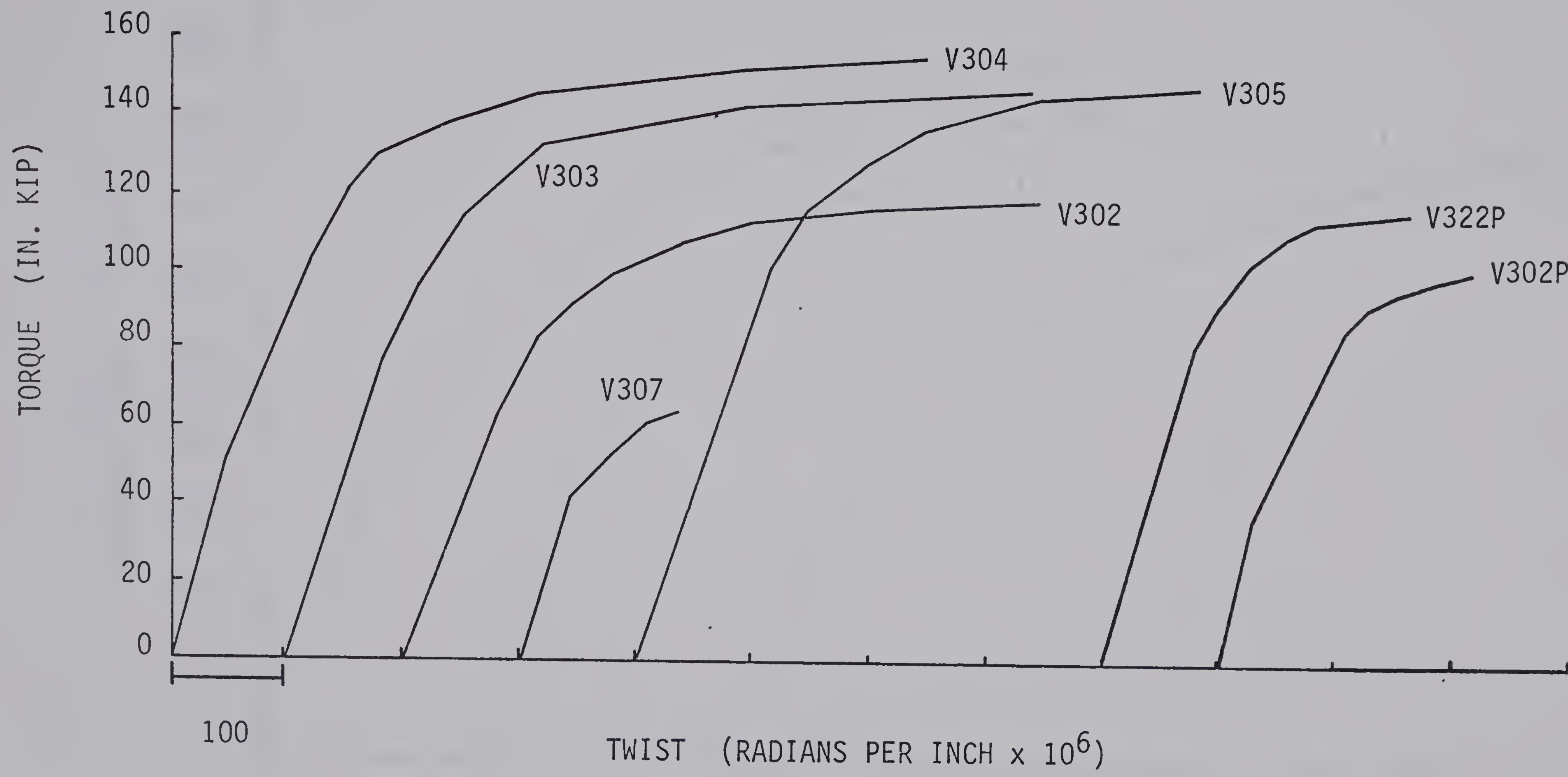

FIGURE A.3 TORQUE - TWIST CURVES (BEAMS V302-V307, V302P, V322P)

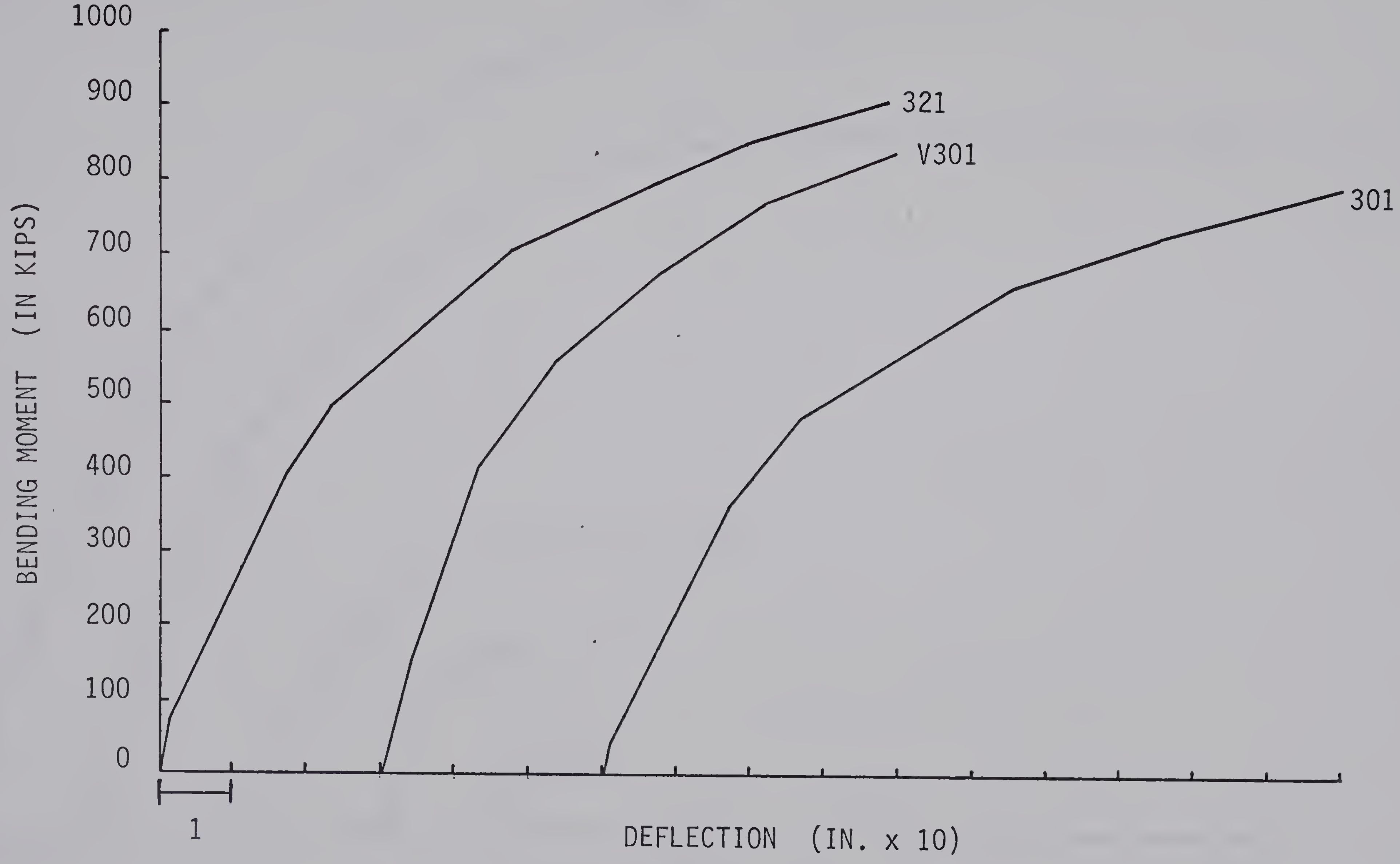

FIGURE A.4 MOMENT DEFLECTION CURVES

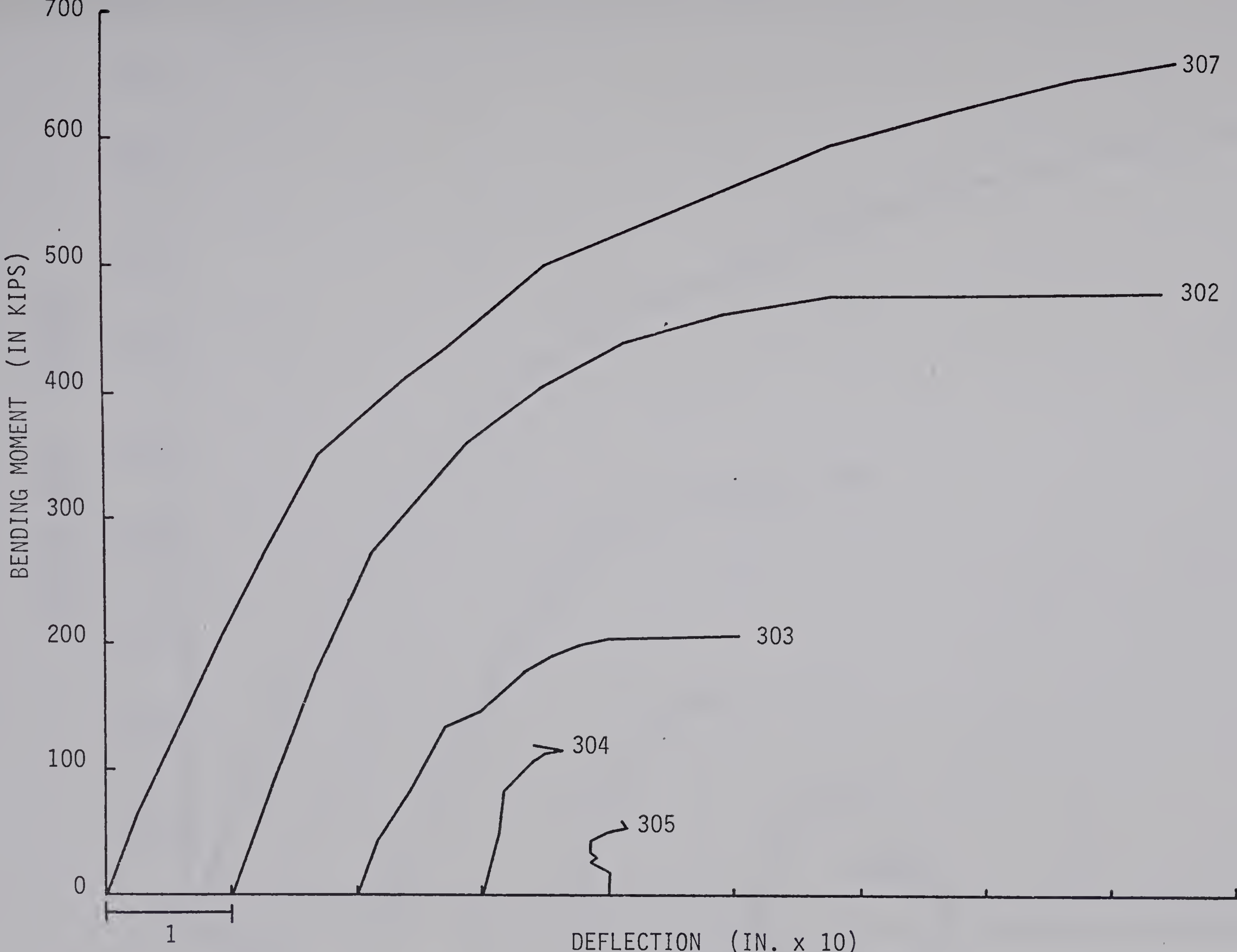

FIGURE A.5 MOMENT DEFLECTION CURVES (BEAMS 302-307)

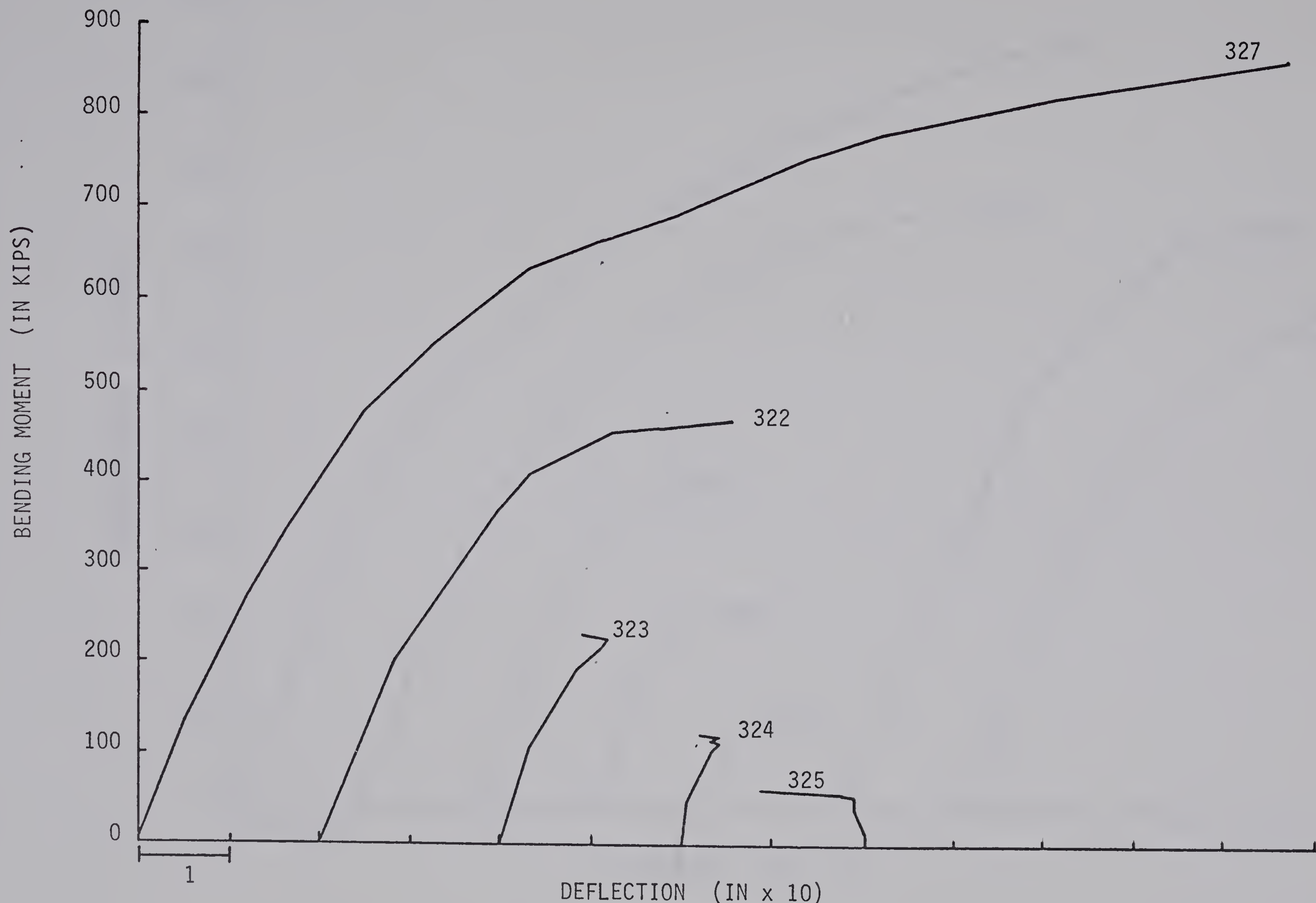

FIGURE A.6 MOMENT DEFLECTION CURVES (BEAMS 322-327)

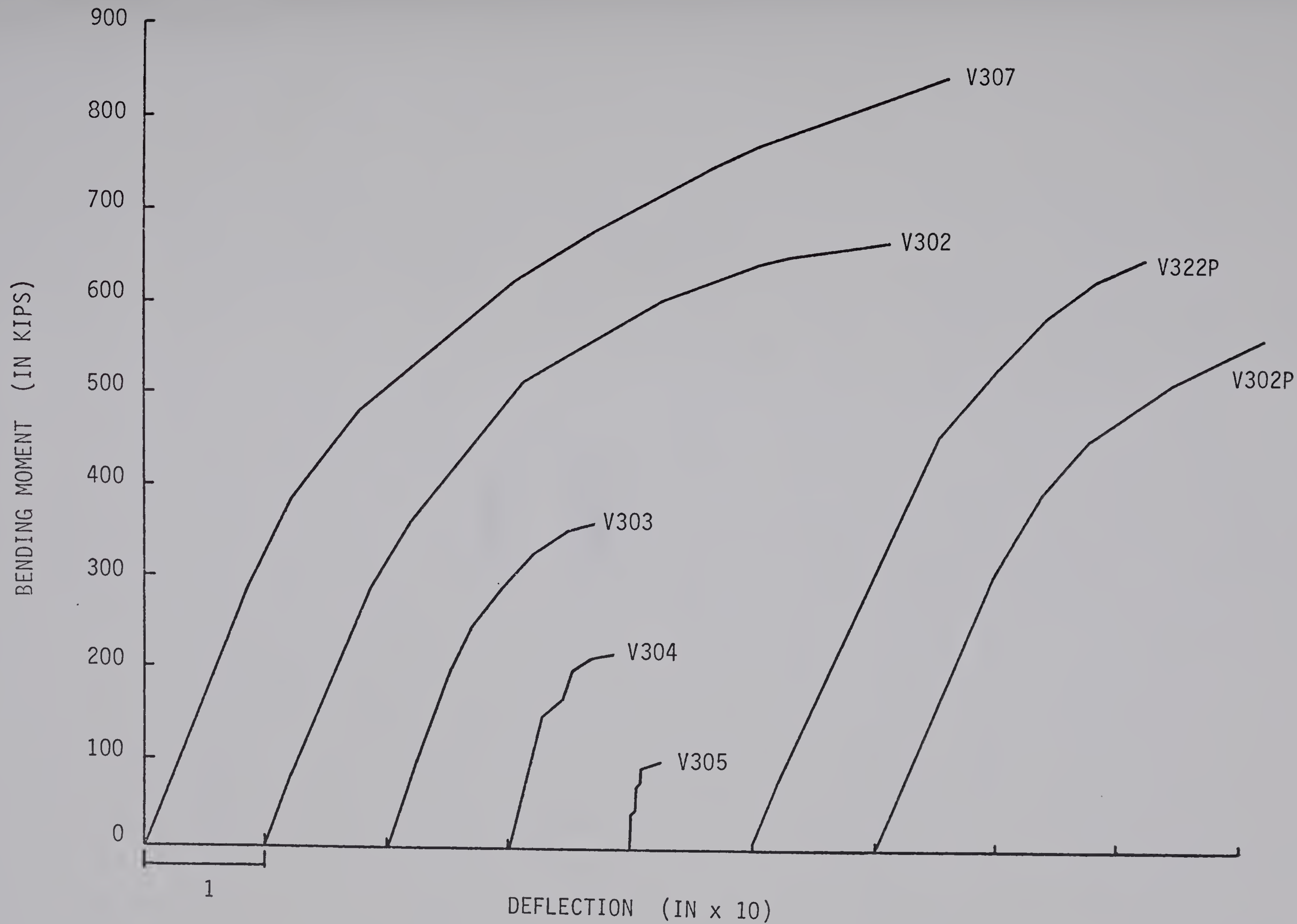

FIGURE A.7 MOMENT - DEFLECTION CURVES (BEAMS V302-V307, V302P, V322P)

APPENDIX B

NOTATION

NOTATIONS

A_p	Total area of prestressing tendon
A_t	Area of one leg of transverse reinforcement
b	Smaller dimension of a rectangular cross section
b_1	Smaller dimension of a rectangular stirrup
d	Larger dimension of a rectangular cross section
d_1	Larger dimension of a rectangular stirrup
e	Eccentricity of prestress from centroid of a cross-section
f_{up}	Ultimate stress of prestressing strand
f_{yt}	Yield stress of transverse reinforcement
f'_c	Cylinder compressive strength of concrete
f'_{sp}	Split cylinder strength of concrete
I_c	Interaction value at cracking
I_u	Interaction value at ultimate
M	Applied bending moment
M_c	Bending moment at first cracking
M_u	Ultimate bending moment
M_{uo}	Ultimate bending moment in pure bending test
P	Effective prestressing force
P_p	Percentage of prestressing strand = $\frac{100A_p}{bd}$
P_t	Percentage of transverse reinforcement = $200(b_1+d_1)A_t$ / bds
s	Spacing of stirrups
T	Applied torsional moment
T_c	Torsional moment at first cracking
T_u	Ultimate torque
T_{uo}	Ultimate torque in pure torsion test

V_u	Ultimate flexural shear
V_{uo}	Ultimate transverse shear in gauge length at failure under bending and shear without the presence of torsional moment
Δ_u	Ultimate deflection
θ_u	Ultimate angle of twist
σ	Average prestress = P/bd